SIGNAL PROCESSING FOR ANTISUBMARINE WARFARE

William Oris Davis

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

SIGNAL PROCESSING FOR ANTISUBMARINE WARFARE

by

William Oris Davis

and

Paul Howard Donaldson

March 1975

Thesis Advisor: George L. Sackman

Approved for public release; distribution unlimited.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Signal Processing for Antisubmarine Warfare		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; March 1975
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) William Oris Davis Paul Howard Donaldson		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE March 1975
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Signal Processing, ASW

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Signal Processing for Antisubmarine Warfare is a short course in electrical signal processing fundamentals and their applications in the field of antisubmarine warfare. It contains an introduction to Fourier transforms and their properties, sampling and quantization, filters and bandwidth requirements, random signals and noise, and an introduction to four types of processing equipment; the DELTIC, energy detectors, correlation detectors,

and beamformers. Course objectives are given in terms of specific questions which a person completing the course should be able to answer. The course text and illustrative material is contained in the appendix to the thesis. The course is designed to be presented in the Fleet to the personnel involved with the operation and employment of detection equipment to provide them a better understanding of the operations accomplished by their equipment and to develop in them a better appreciation of the problems and limitations associated with signal detection in the antisubmarine warfare environment.

Signal Processing for Antisubmarine Warfare

by

William Oris Davis
Lieutenant, United States Navy
B.S., West Virginia Institute of Technology, 1968

Paul Howard Donaldson
Lieutenant, United States Navy
B.S., United States Naval Academy, 1966

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN SYSTEMS TECHNOLOGY

from the

NAVAL POSTGRADUATE SCHOOL
March 1975

ABSTRACT

Signal Processing for Antisubmarine Warfare is a short course in electrical signal processing fundamentals and their applications in the field of antisubmarine warfare. It contains an introduction to Fourier transforms and their properties, sampling and quantization, filters and bandwidth requirements, random signals and noise, and an introduction to four types of processing equipment; the DELTIC, energy detectors, correlation detectors, and beamformers. Course objectives are given in terms of specific questions which a person completing the course should be able to answer. The course text and illustrative material is contained in the appendix to the thesis. The course is designed to be presented in the Fleet to the personnel involved with the operation and employment of detection equipment to provide them a better understanding of the operations accomplished by their equipment and to develop in them a better appreciation of the problems and limitations associated with signal detection in the antisubmarine warfare environment.

TABLE OF CONTENTS

INTRODUCTION

Signal Processing for ASW was developed as an introductory course intended for the Naval Officer who has had the traditional training in ASW hardware operations and ASW administrative requirements. However, it is applicable to anyone involved in ASW - officer, enlisted, or civilian - who does not have an engineering background.

There are several fields that are of particular interest to the ASW specialist: acoustics, signal processing, environmental sciences, and probability/statistics. This course addresses only the signal processing aspects of ASW. Short courses covering the other three areas are also under development at the Naval Postgraduate School. Other theses relevant to these areas are KING and SWIFT, 1975; DECKER and SOLLENBERGER, 1975; and SMITH and VICE, 1975. For anyone who takes all four of these courses, it is recommended that the signal processing be taken last, because the information contained in the other three will enhance the student's ability to obtain an understanding of the capabilities and limitations of signal processing.

COURSE OBJECTIVES

The overall objective of this course is to familiarize the student with the principles of signal processing used in ASW sensor systems. The student who successfully completes this course will be knowledgable in the specific areas which follow to the degree indicated.

A. SIGNAL PROCESSING THEORY

Understand the theory of signal processing sufficiently well to do the following.

1. Explain the relationship of the Fourier Transform to the "time domain" and the "frequency domain".

2. Given a Fourier Transform pair, identify the operations they express.

3. Given a square wave and a rectangular wave:

a. sketch the correlation of one with the other;

b. sketch the convolution of one with the other;

c. transform the given waves from the time domain to the frequency domain, multiply them, and apply the inverse transform to return to the time domain;

d. compare the results obtained in a, b, and c.

B. SIGNAL PROCESSING FUNDAMENTALS

Be sufficiently knowledgable of signal processing fundamentals to do the following.

1. Show the difference between a digital and an analog signal.

2. Discuss how noise limits quantization in the sampling and quantizing of a signal.

3. Relate Sampling Theorem, Nyquist Rate, and "aliasing".

4. Given a plot of signal and noise as power density versus frequency, show how filtering increases the ratio of signal power to noise power.

5. Given a plot of filter response versus frequency, identify a high pass, a low pass, and a bandpass filter.

6. Relate filter bandpass, integration time, and frequency resolution.

7. Define the statistical characteristics of a random signal and relate them to the signal voltage (or current) and power components.

8. List the assumptions necessary to apply statistical techniques to the processing of random signals.

9. Contrast the cross correlation of correlated signals with the cross correlation of uncorrelated signals. Show how the correlated signals are enhanced.

10. Explain the three effects of filtering gaussian white noise. Discuss the relationship of these effects when processing signal and noise.

C. DELTIC (DElay TIme Compressor)

Be able to describe the DELTIC, including:

1. how the DELTIC functions;

2. the benefits of using a DELTIC in the processing of a signal;

3. given the specifications of a particular DELTIC, calculate the integration time required and the frequency resolution of the output.

D. ENERGY DETECTION

Describe energy detection by explaining:

1. how the system determines that a target is present;

2. how the threshold setting affects the probability of dedection, P(d), and the probability of false alarm, P(fa);

3. the significance of the direct relationship of a P(fa) with a given P(d) as represented by a Receiver Operating Characteristics (ROC) curve.

E. CORRELATION DETECTION

Be able to:

1. describe the method used to discriminate between the returning echo and noise;

2. describe the effect of doppler on correlation;

3. explain how doppler is determined;

4. describe how pulse length effects doppler resolution;

5. describe how pulse length effects range resolution;

6. list the advantages of using an FM slide;

7. state the advantages and disadvantages of using Pseudo-Random Noise pulses for search and tracking;

8. given a two-hydrophone array being used with a correlation detector, sketch and explain how the signal direction is obtained, why a direction ambiguity exists, and how the ambiguity may be resolved;

9. explain the limitations on the type of signals which may be processed passively by a correlation detector, and why these limitations exist.

F. BEAM FORMING

1. Describe the method used by the beam former to obtain direction.

2. Name the type detector used with a passive beam former.

3. Given a hydrophone array in a beam former configuration, sketch the output of the beam former for a signal in the beam and a signal out of the beam. Show by comparing the outputs the importance of setting the threshold to determine a target present in the beam.

THE STUDENT

It is expected that the student will be currently involved in ASW. Prior courses in probability, college algebra and calculus, and basic electricity are helpful but not required, since the course is self-contained.

THE INSTRUCTOR

There are three basic qualification needed by the instructor for this course: a background in electrical engineering that includes the fundamentals of signal processing; a knowledge of current practices in ASW; the ability to teach. The Naval Postgraduate School has faculty available with these

qualifications, and therefore should be considered as a logical source. NPS graduates may also provide a source of people with the requisite qualifications.

COURSE DEVELOPMENT AND PRESENTATION

This course was designed to provide a thorough understanding of the principles involved in the application of spectrum analysis to ASW signal processing, and the operation of the DELTIC, energy detectors, correlation detectors, and beam formers. These processing methods comprise the core around which the U.S.NAVY's ASW sensor systems are built. In proceeding toward the above goal, the course follows a logical approach of building from the basics of Fourier Transforms and Fourier Transform Properties. Following those two sections is a section on Sampling and Quantization and a section on Filters and Bandwidth Requirements. These latter two sections may be interchanged, but both are required for the section on Random Signal, Power Spectral Density, and Noise. The methods of processing are presented last; in order they are, DELTIC, Energy Detection, Correlation Detection, and Beam Forming. Where appropriate, specific hardware has been mentioned, but it is emphasized that the course is primarily devoted to principles and methods that may be applied to various systems employed on either air, surface, or subsurface platforms.

This course is designed to be taught in the Fleet. Although it requires no outside references, each student

should be provided with a complete copy of the course material for his (her) use and retention. The course objectives are very specific. They are designed for use by the instructor and student as a guide for measuring achievement: they indicate to the student what he should learn; they should serve the instructor as the basis for examination questions. The only training aid required by the instructor is a chalk board, as students may follow along on the illustrations in the course material. It will be helpful if 35 mm slides of all the illustrations are available. Overhead projectors, although common at shore stations, are not common on board ship, whereas a 35 mm slide projector is normally on board for other training requirements. References to books which cover the subjects in greater detail, and which have been of assistance to the authors in the preparation of this course are included in the bibliography.

If the course is presented independently, no more than two hours per day should be devoted to in-class presentation. The student should plan to spend a minimum of one hour of review and study for each in-class hour. The student must have time for the information to become impressed in his memory and become familiar to him. If all four courses are presented concurrently (feasible in a training environment due to the availavility of the students time and approximately the same time required for each course), no more than one in-class hour per day should be spent on any one course. Regardless of the method used, the instructor should be available

at other than scheduled-class time to clarify points on which the student may have questions.

The course is designed for twenty in-class hours. The in-class time requirement schedule is as follows.

SECTION	HOURS
Introduction to Fourier Transforms	4
Fourier Transform Properties	4
Sampling and Quantization	1
Random Signal, Power Spectral Density, and Noise	4
DELTIC	1
Energy Detection	2
Correlation Detection	2
Beam Forming	1

The material covered is equivalent to that covered in a two-credit hour, one quarter course at NPS. The sections which required the most time to develop were the two sections concerning Fourier transforms and the section on Random signal, power spectral density, and noise. In these sections the emphasis is on development of theory and concept which proved to be the most difficult to explain clearly and in a manner to preclude misunderstanding. The other areas were easier to address as they lend themselves much more readily to a descriptive explanation.

CONCLUSIONS

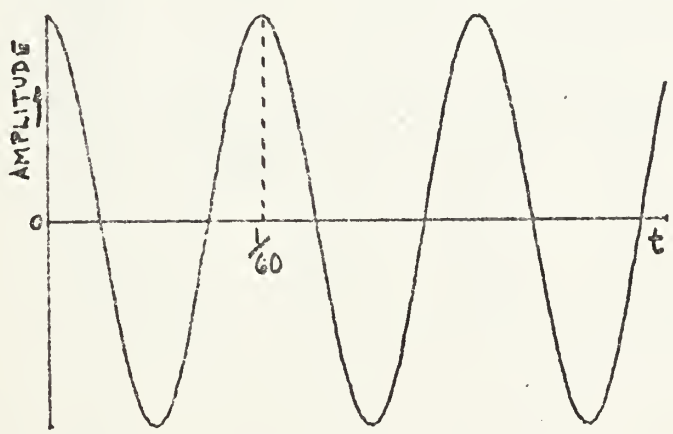
This course effectively covers the major areas of signal processing applicable to ASW. Its development has been directed toward the understanding of theories and principles used in current ASW signal processing methods. These theories and principles will remain, although in future processing methods the relative importance of their individual applications may be altered. For example, the current trend in ASW signal processing is toward more utilization of digital processing methods. This will increase the importance of thoroughly understanding sampling and quantization.

An area which remains to be addressed is the use of digital computers in signal processing methods. This should include an introduction to boolean algebra, elementary logic and set theory, the use of truth tables and Karnaugh maps, binary arithmetic and electronic logic circuits. It is recommended that such a course be developed as part of the continuing education program at the Naval Postgraduate School.

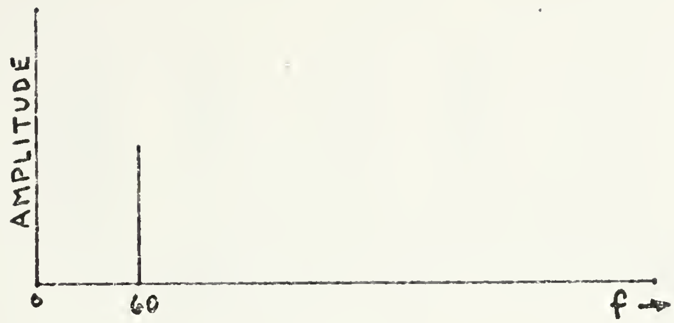
APPENDIX

I. INTRODUCTION TO FOURIER TRANSFORMS

An introduction to signal processing requires, for understanding of many processing schemes, a working familiarity with Fourier transforms. To gain an understanding of what Fourier transforms are, what they can do, and how to use them, let us first examine a common type of signal. The electrical power used in the United States is 60 Hertz (cycles per second) alternating current. That is, the current changes its direction of flow in such a manner as to complete a cycle sixty times per second. If we plot the amplitude of the current in a given direction as a function of time, the result is a sinusoid.

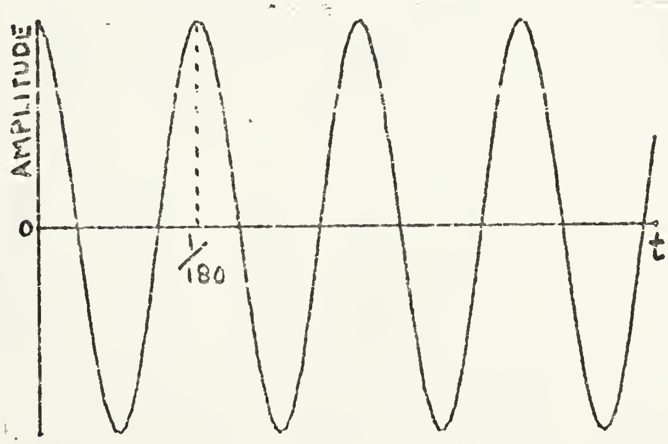


Since this is a description of the signal as a function of time, it is said to be the "time domain" description. If we plot the signal as a function of frequency, we get the "frequency domain" description.



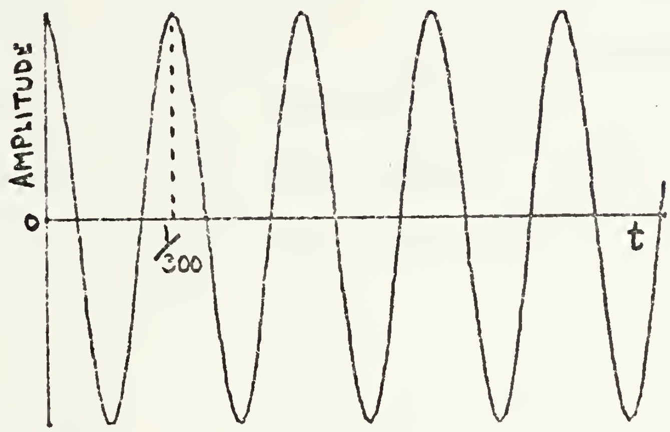
Since the signal consists of a single frequency, there is but one frequency represented on the f-domain plot.

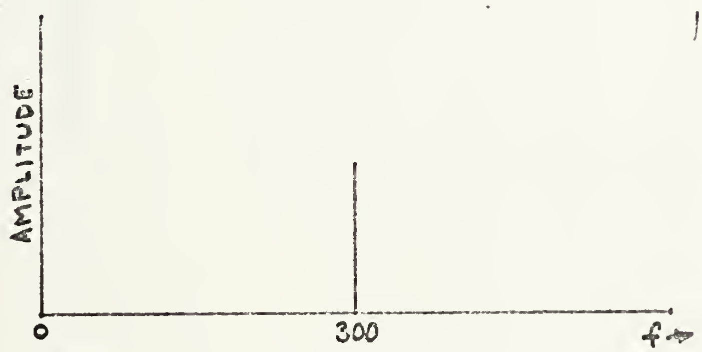
Let us now look at a different signal, say an alternating signal of 180 Hz. The time domain and frequency domain plots are as follows.

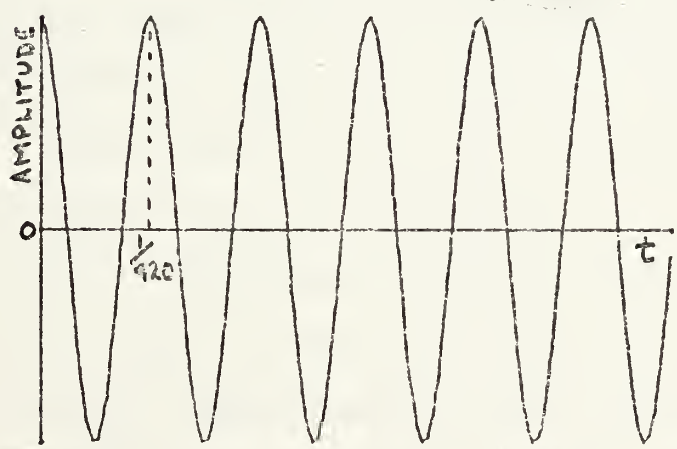


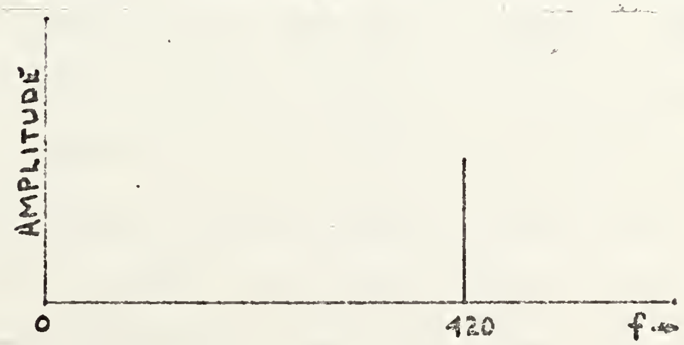
Note the way in which the changes in the signal are evidenced, both in the time domain and the frequency domain.

Likewise, two other signals, 300 Hz and 420 Hz.



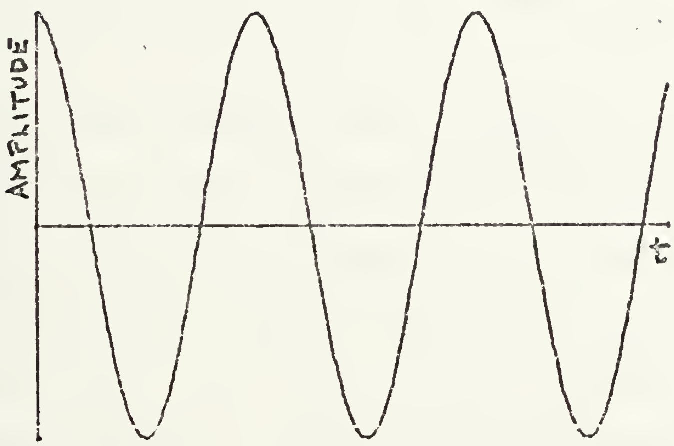




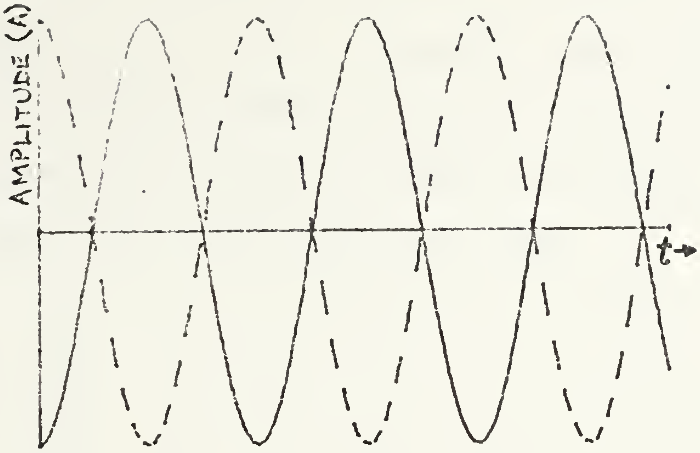


So far we have looked at signals of a single frequency. We are going to look at signals made up of many frequency components, but first let us review the concept of phase.

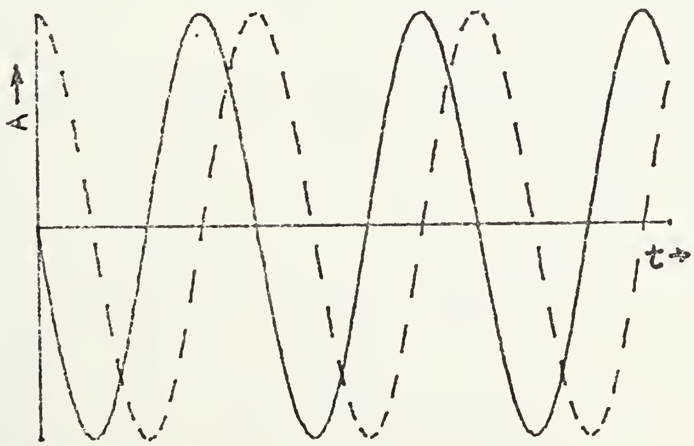
PHASE: The phase of a sinusoidal signal with respect to a reference signal is the relationship between corresponding parts of their cycles in the time domain. For example, take a signal for reference.



Comparing a signal to this reference we can determine the phase of the new signal with respect to the reference. If the new signal crosses the t axis at the same time and in the same direction as the reference, it is said to be "in phase" with the reference, or to have a phase angle of 0°. If the new signal crosses the t axis on the down-swing when the reference signal crosses on the up-swing, as shown here, it is said to be "180° out of phase". The reason for using "180° will be shown later in this development. For the moment, however, just accept the fact that a signal is said to go through a phase angle of 360° each complete cycle. The phase of a signal can either lead or lag the reference. That is, the signal may cross the t-axis on the down-swing

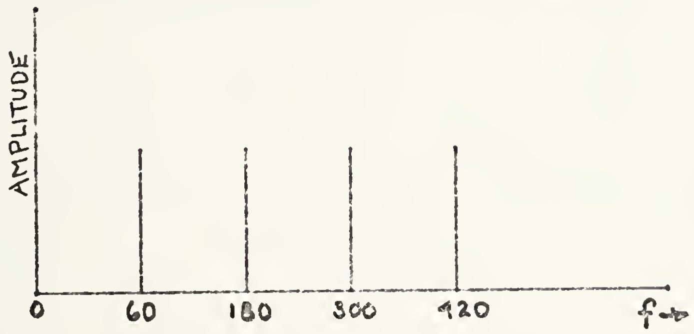


before or after the reference does. The first case would be leading and the second, lagging with respect to the reference. Because of this, a signal is not generally described as having a phase angle greater than 180° since a 180° leading signal looks just the same as a 180° lagging signal. For example, a signal which is leading the reference by 270° looks the same as one lagging the reference by 90°, and is usually described as the latter.

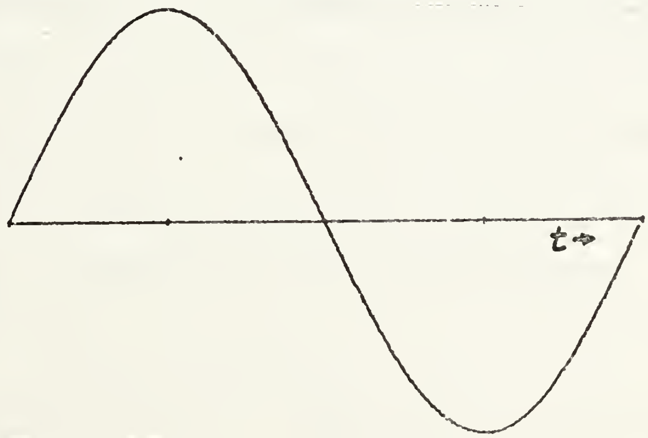


Signals with Multiple Frequency Components: Returning to the signal with more than one frequency component, let us look at the way in which this would be represented

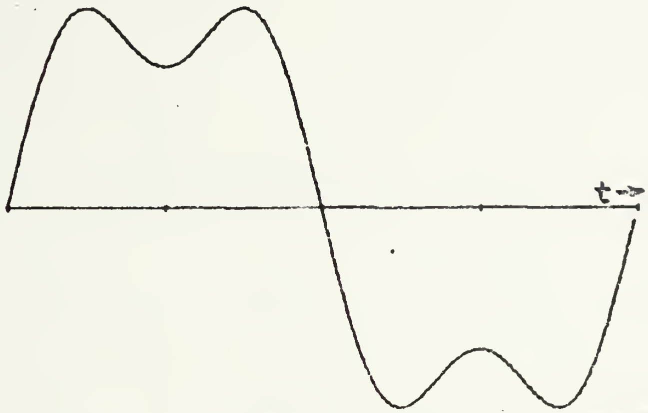
in the time and frequency domains. Take, for example, the four frequencies we looked at earlier: 60, 180, 300, and 420 Hz. If we had a signal which was composed of equal amplitude components of each of these frequencies, the frequency domain plot would look like this.

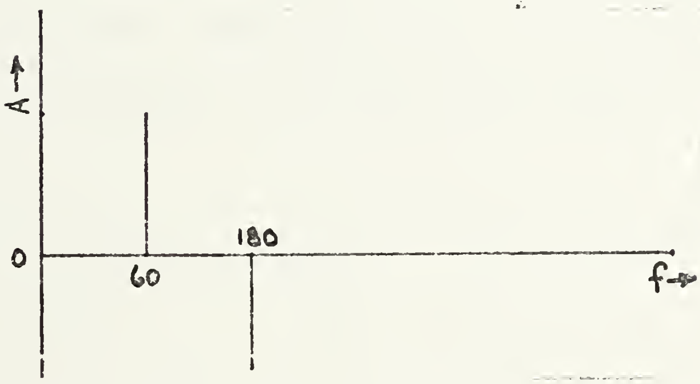


Now let us look at the time domain plot of this signal, starting with just the 60 Hz frequency present and then as we add to the signal judiciously chosen amplitude and phase components of the other frequencies. First, take the 60 Hz. portion.

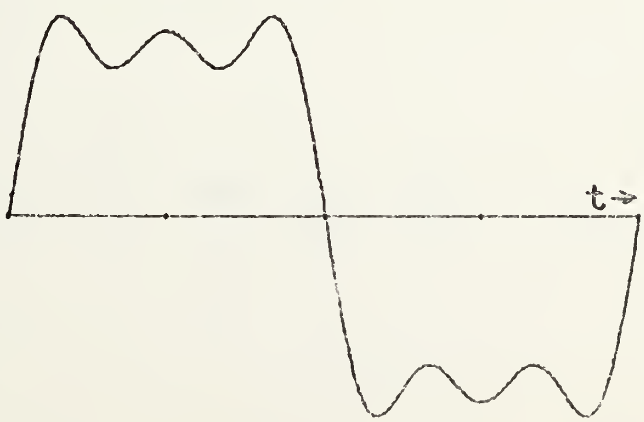


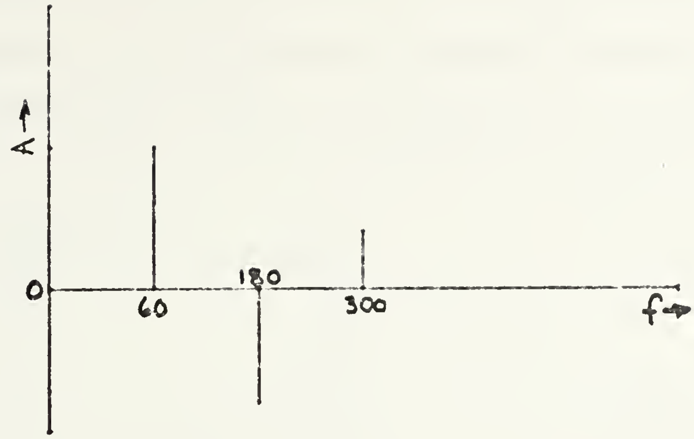
Now add one-third the amplitude of the 180 Hz signal but shift it 180° out of phase with the 60 Hz signal.



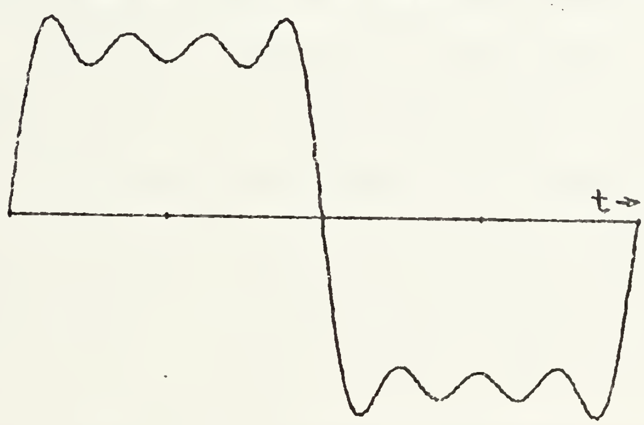


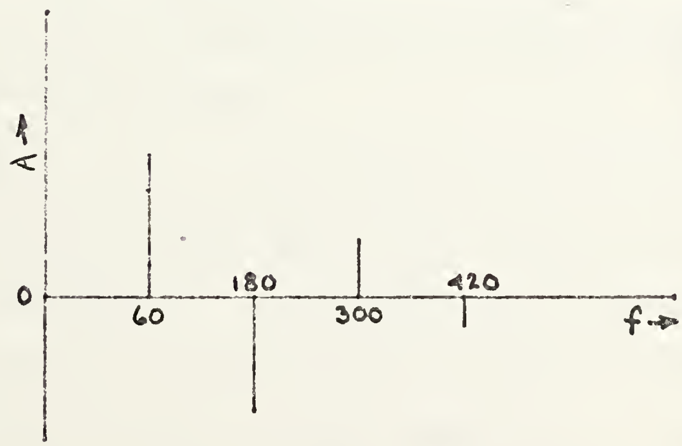
Note the fact that the 180 Hz component was added 180° out of phase shows up on the frequency domain plot as a negative amplitude. The reason for this will become apparent shortly. For now, however, let us continue with the other frequencies. To the signal we now add one-fifth the amplitude of the 300 Hz component, in phase with the 60 Hz signal.



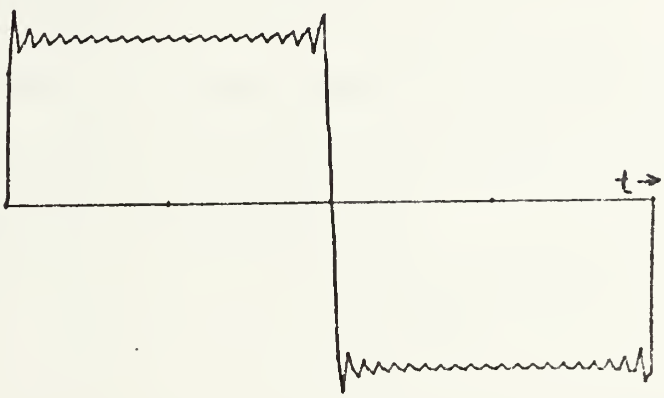


Now add one-seventh the amplitude of the 420 Hz component, but 180° out of phase from the 60 Hz signal.





If we continue to add carefully selected frequency components to the signal, the time domain signal becomes more and more like a square wave. With 20 components, the wave looks like this.

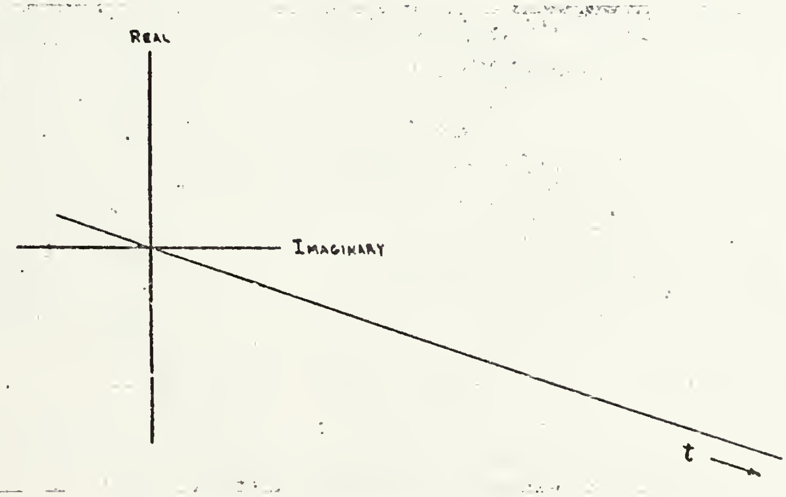


From this, one can see that by careful choice of component frequencies, and their respective amplitudes and phases, that a square wave can be generated from purely sinusoidal components. If an infinite number of components were used, the resulting waveform would be a perfect square wave. This concept is the basis on which a large part of our signal processing theory rests. There is not time in this course to demonstrate it, but it is a basic theorem of Fourier analysis that any shape waveform can be created from combinations of pure sinusoidal signals.

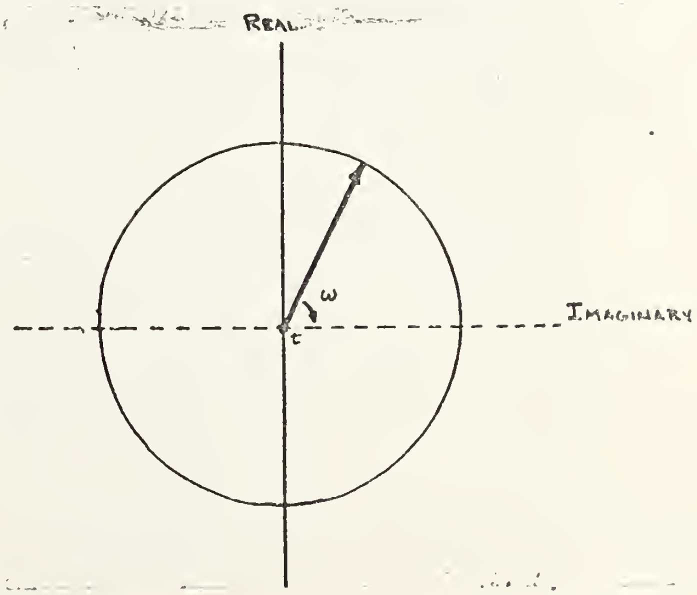
Complex Quantities: In order to simplify the mathematics involved later in the course, we must introduce the concept of complex quantities. In solving the mathematical equations describing waves, one encounters solutions which contain terms multiplied by $\sqrt{-1}$. As one can readily see, there is no real number which, when multiplied by itself, will give a number

which is negative. Thus, these quantities are known as "imaginary". They are, in actuality, artifacts of the mathematics and are not directly related to tangible quantities, however, they prove to be of use in describing the behavior of wave phenomena.

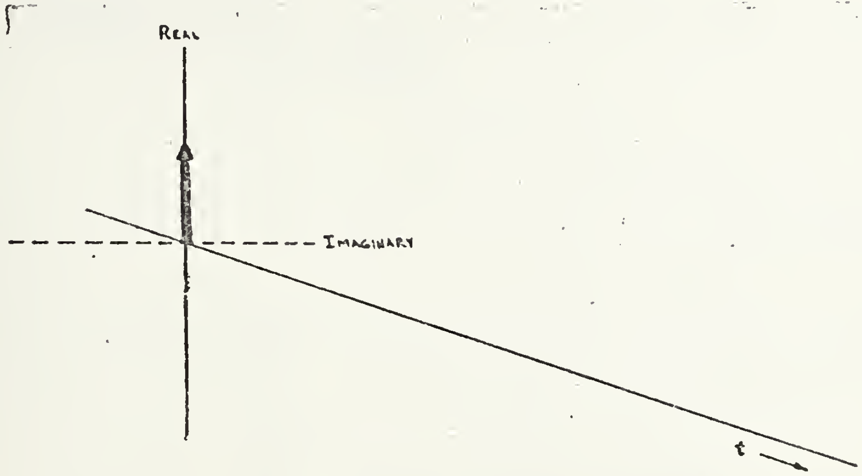
If we plot all "real" quantities on one axis, all "imaginary" quantities on another, and time on a third axis, we get a coordinate system like so:

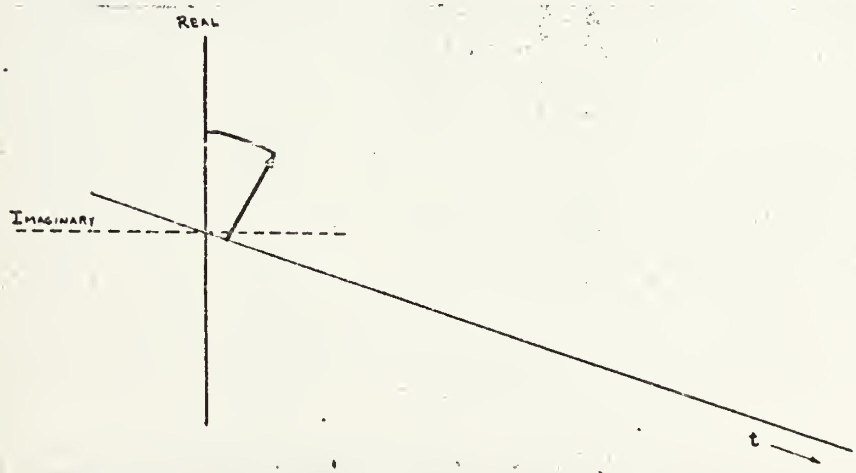


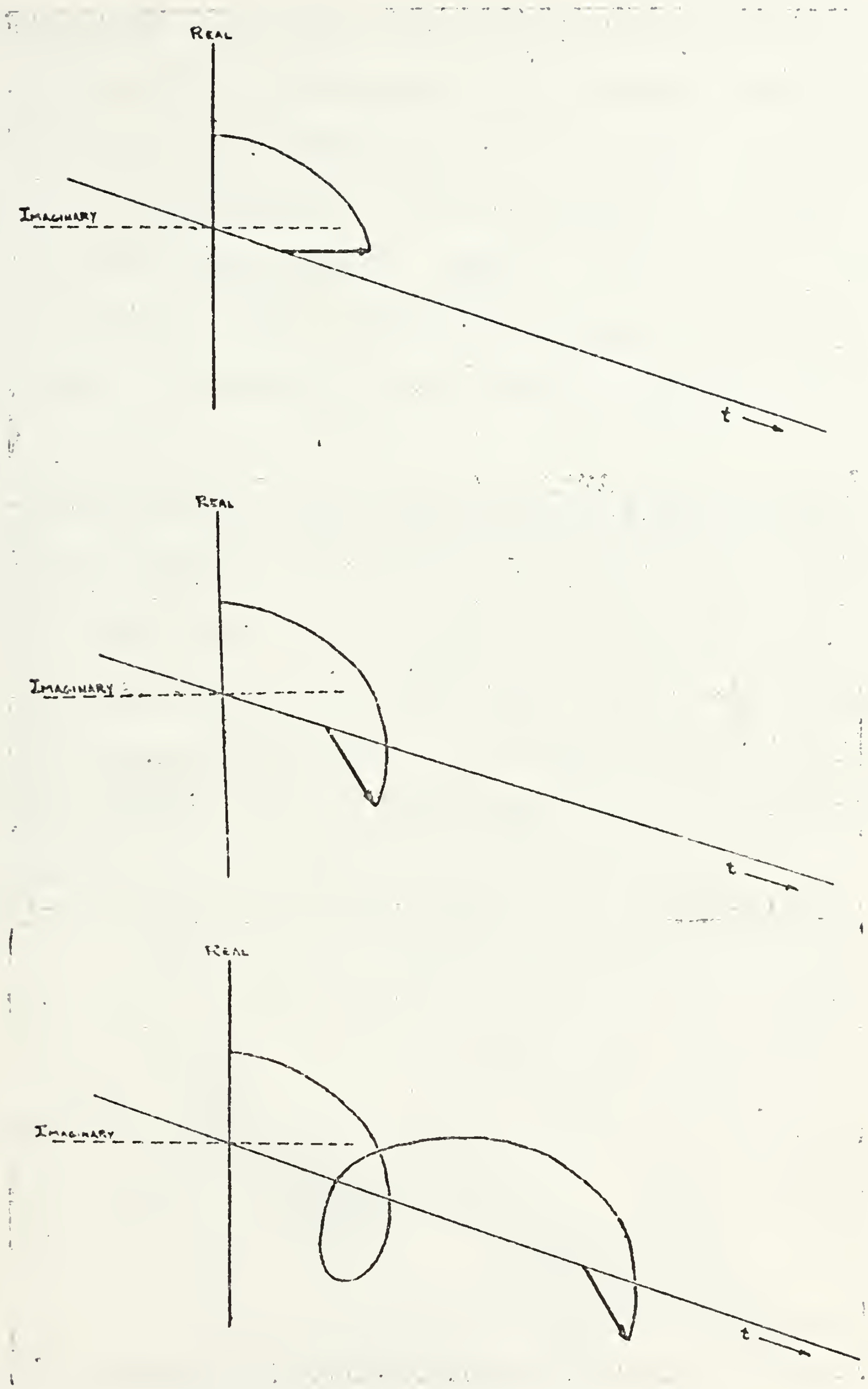
One of the results of the solutions described above is that a sinusoidal signal in the real plane (the plane formed by the real and time axes can be generated by a vector rotating with a uniform angular velocity in the complex plane (formed by the real and imaginary axes). Looking at this vector down the time axis shows:



As this vector rotates it is also moving down the time axis, so the path swept out by the tip of the vector describes a helical path around the time axis.



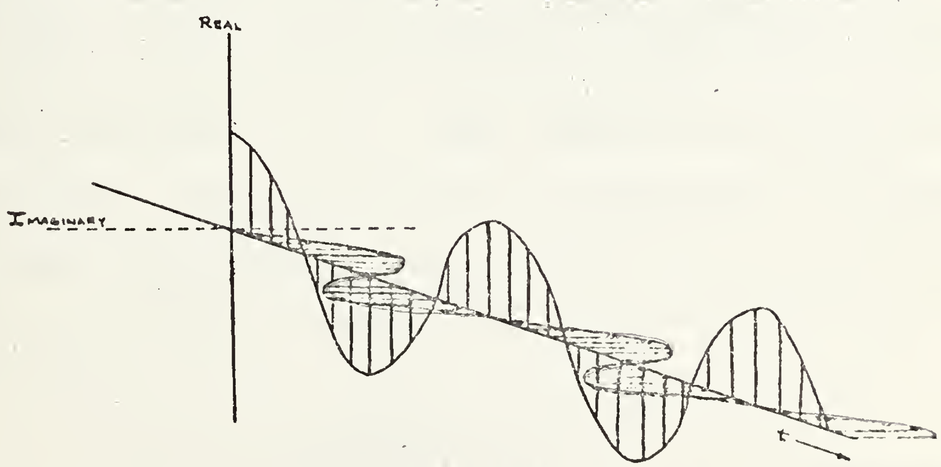




At this point it should be noted that there is another measure of angular rotation known as the "radian". There are 2π radians in $360°$. This measure is used because it simplifies the mathematics involved in solving wave equations.

Returning to the vector generator, we see that the number of times it makes a complete 360° (2π radians) sweep in one second of time is the frequency of the signal. The rotational velocity of the vector, ω, is the number of radians swept out by the vector per second. Thus $\omega = 2\pi f$ since it sweeps out 2π radians each 360° rotation and makes f rotations per second. Thus ω is known as the "radian frequency" of the signal.

If we look at the projections of the vector on the real and imaginary planes as it rotates, we find that the projection on the real plane is a cosine wave and the projection on the imaginary plane is a sine wave. Also, we see that the vector rotates 360° during one cycle of the cosine wave on the real plane. This is where the use of 360° of phase angle arose.



The vector has components in the real and imaginary planes. A mathematician, by the name of Euler, has shown the relationship:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta,$$

where j is taken to mean "in the imaginary plane".

By vector addition, the vector generating our signal is the vector sum of the real component, cos ωt, and the imaginary component, j sin ωt.

$$\text{Vector} = \cos \omega t + j \sin \omega t$$

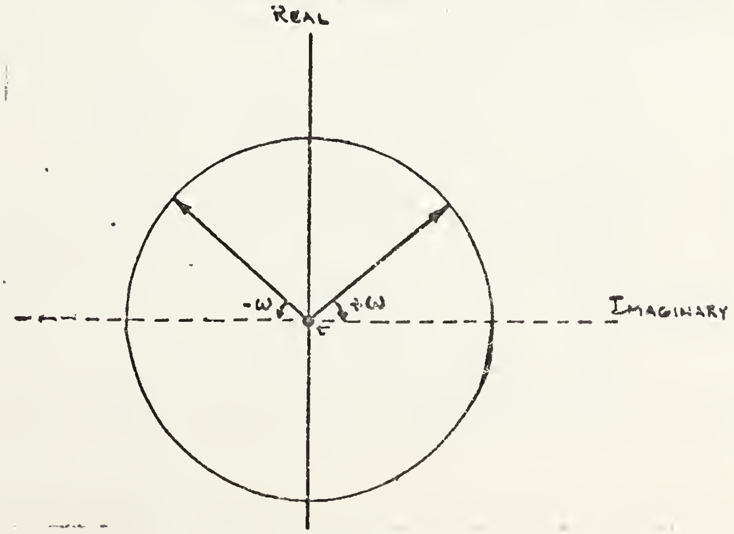
We see that this is of the form of Euler's identity if $\theta = \omega t$. Thus, the mathematical description of the vector is $e^{\pm j \omega t}$. This vector is known as a "phasor".

We have seen that cos ωt is the projection of the vector tip path on the real plane. Solving Euler's identity for cos ωt, we get:

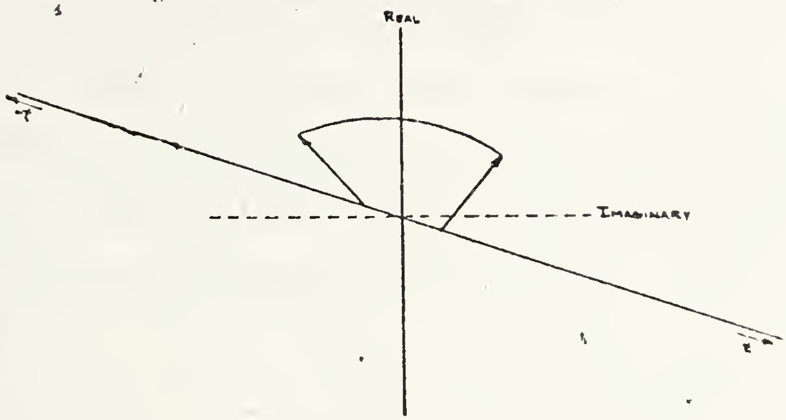
$$\begin{aligned}\text{Cos } \omega t &= \text{Real Part of } [e^{\pm j \omega t}] \\ &= \text{Re}[e^{\pm j \omega t}].\end{aligned}$$

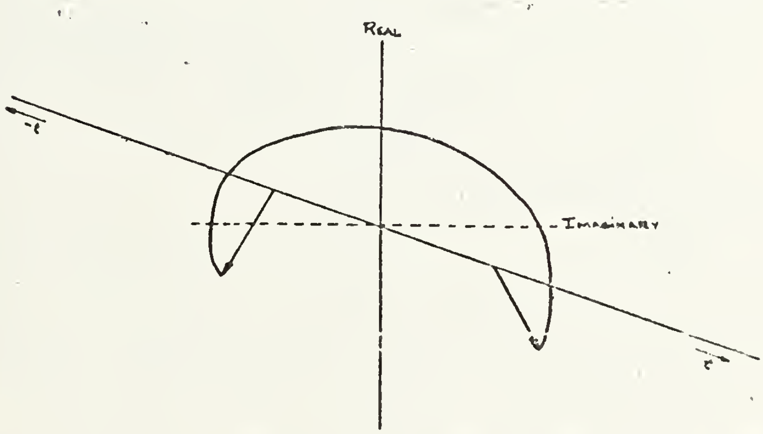
[The convention of using $e^{\pm j\omega t}$ for cos ωt is used in EE because it suggests several properties of the cosine not apparent in the written notation "cosine ωt".]

What does this mean? The ± sign indicates that there are two solutions. This corresponds to two phasors rotating in the $+\omega$ and $-\omega$ directions, respectively.

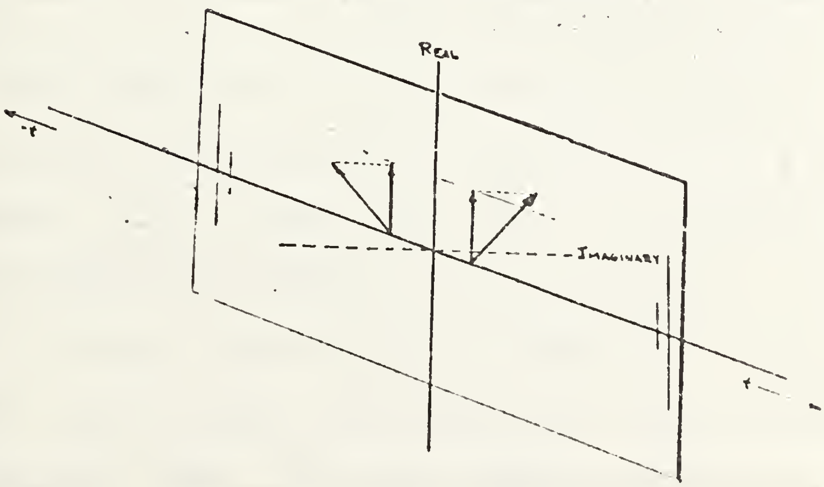


These phasors unwind in opposite directions along the t-axis.





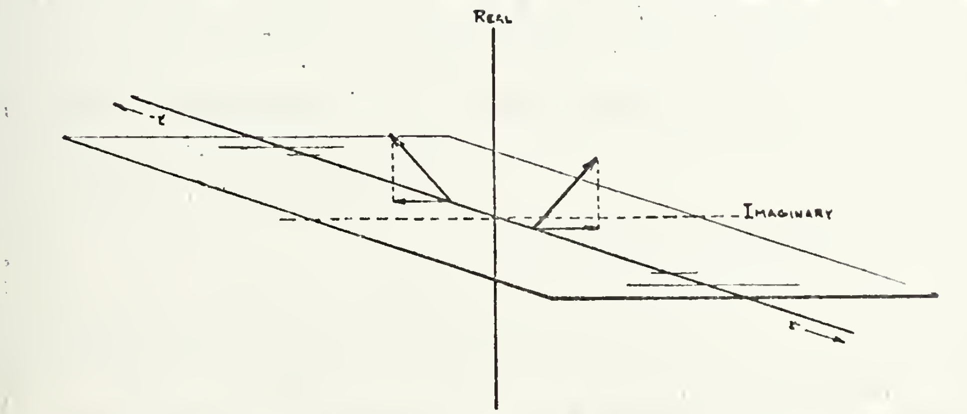
Looking at the projection of these phasors on the real plane, we see cosωt.



Solving Euler's identity for Cos ωt, we obtain

$$\text{Cos } \omega t = \frac{e^{j\,\omega t} + e^{-j\,\omega t}}{2} .$$

We can see this, as the two contributions of the phasors always add in the same direction on the real plane, therefore, the + sign in the formula. Looking at the projection of these phasors on the imaginary plane, we see sin ωt.



Solving Euler's identity for Sin ωt, we obtain

$$\text{Sin } \omega t = \frac{e^{j\,\omega t} - e^{-j\,\omega t}}{2j} .$$

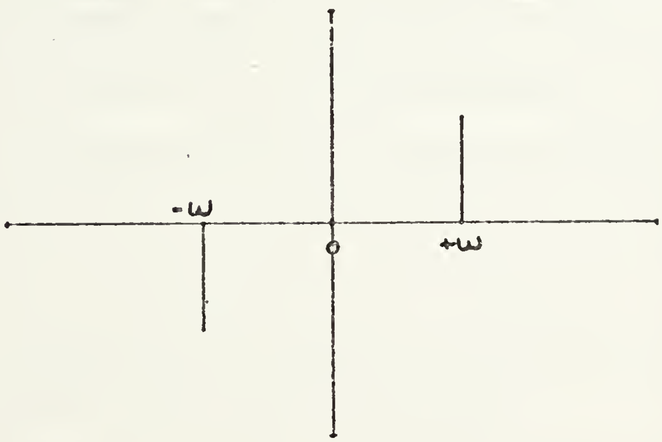
Because the two projections are always in opposite directions on the imaginary plane, we get the – sign in the formula.

We have shown how a single frequency plots as a single line in the frequency domain. This relates to a line for each phasor. But we have seen that the mathematical solutions establish two phasors for each frequency, one rotating at +ω and one rotating at -ω. Thus, in actuality, we have been showing only one side of the spectrum, the positive frequency side. The full spectrum of cos ωt is then:

The lines are both on the same side of the ω axis because of the + sign in the formula

$$\text{Cos } \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2} .$$

The full spectrum of sin ωt is,



Note that the lines are on opposite sides of the ω axis due to the – sign in the formula

$$\text{Sin } \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} .$$

Note also that the lines correspond to projections on the real plane for cos ωt and the imaginary plane for sin ωt.

We have previously shown that arbitrary waveforms can

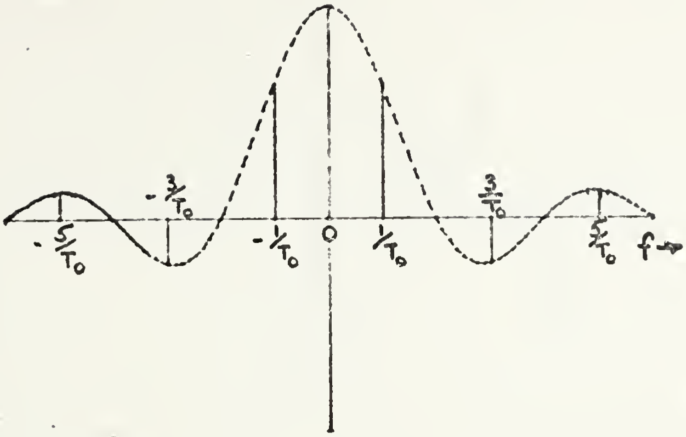
be made from combinations of sines and cosines of the proper amplitudes, phases, and frequencies. How do we determine which frequencies and what amplitudes and phases to use? In effect, we correlate the waveform with various frequencies. We compare the waveform to a given frequency and see how well they match. One measure of this is the point by point product of the two averaged over a period. This measure, known as C_n (n^{th} frequency coefficient) is the following:

$$C_n = \frac{1}{T_o} \int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} x(t)\, e^{-j\omega_n t}\, dt$$

where x(t) is the waveform of interest, $e^{-j\omega_n t}$ is the cosine wave being compared, and the integral serves to sum the product over the period, T_o, of the waveform. This determines how closely the waveform matches a given frequency. In order to find the components of a given waveform, we must compare the waveform to all frequencies. The "spectrum" of the waveform is then the summation of C_n's multiplied by the frequency component ($e^{j\omega_n t}$) over all frequencies.

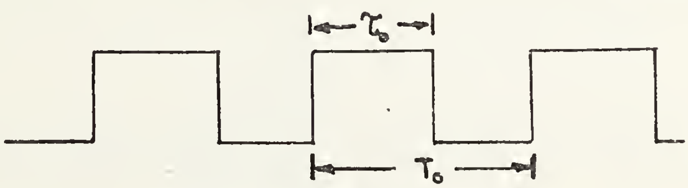
$$X(f) = \sum_{n=-\infty}^{\infty} C_n\, e^{j\omega_n t}$$

Using this technique, to analyze a square wave train, we get the spectrum as shown on the following page.

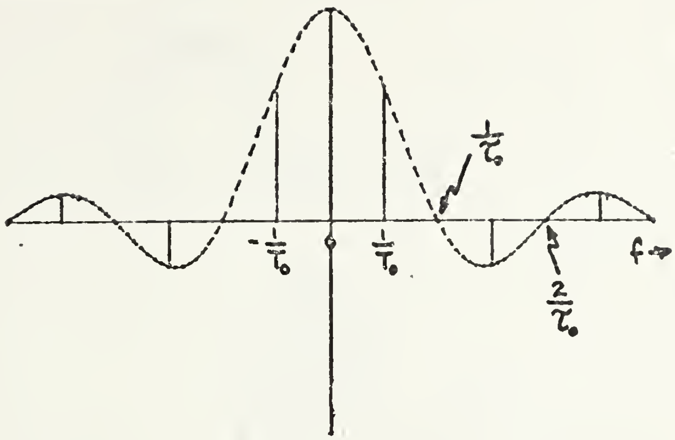


We find that all the C_n's equal zero except for a few. These turn out to be harmonics of the lowest component. This is because the signal is compared to sines and cosines. If the signal is compared to other forms of signals, the C_n's will not necessarily be harmonics. This technique of analysis of waveforms was developed by Fourier and thus has become known as Fourier Analysis. The fact that some of the C_n's are negative indicates that the frequency corresponding to that C_n is to be 180° out of phase with the positive sign C_n frequencies.

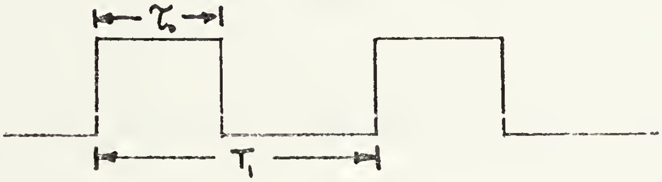
Let us look at some square wave pulse trains and analyze their spectra.

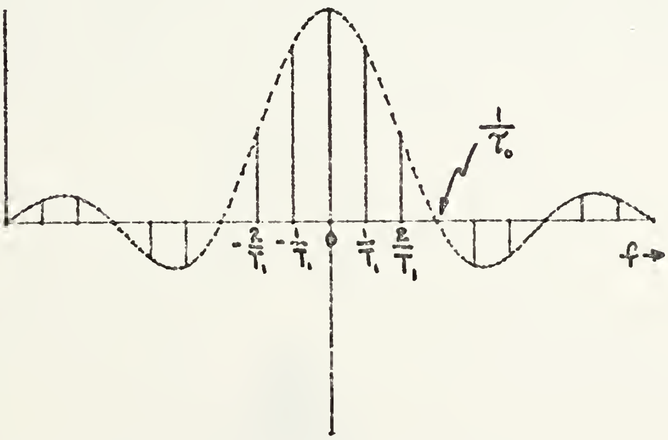


If we solve for the C_n's and plot we get the curve as shown on the following page.



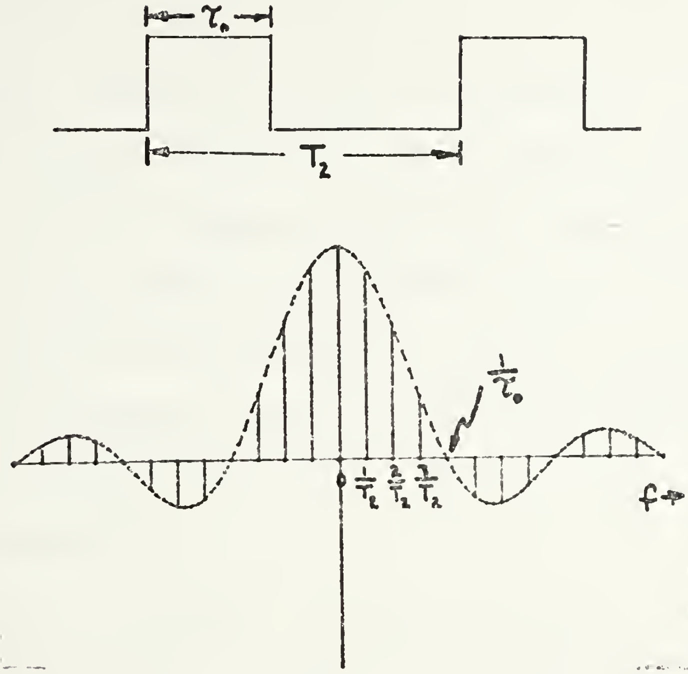
If the period of the pulse train is increased but the length of each individual pulse is kept constant, the following curves are obtained.



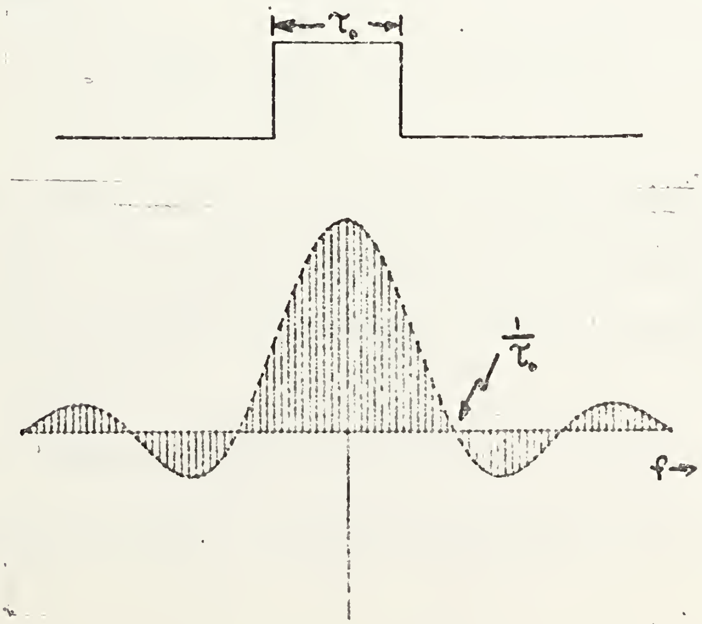


Note that the point where the envelope of the C_n's crosses zero remains the same, but the number of C_n's increases. That is, there are more frequency components present in this signal than there were in the first. Let us now increase the

period even more, still keeping the pulse length constant.



We see that the zero crossings of the envelope remain constant, but the number of C_n's increases again. If we were to increase the period to the point where there was only one square pulse left, that is, $T \to \infty$, we would find that the number of C_n's would be infinite, but the zero crossings of the envelope would remain the same.



From these examples we can see that the zero crossings interval of the envelope is related to the inverse of the pulse width, and the interval between C_n's is related to the inverse of the period of the signal. This, and other properties of the Fourier relations, will be discussed in greater detail later in the course.

The form of the envelope of the C_n's for the square wave appears so frequently that it has been given a name of its own. The form is (sin x)/x, which is called "sinc" (pronounced "sin-cee").

Fourier Transforms: We have shown that as the period of a signal becomes infinite, its number of spectral components also becomes infinite. In this case, the spectrum of a signal can be characterized by the formula of the envelope of the C_n's. Techniques have been developed for finding the formula of the envelope without solving for the C_n's and one method of doing this is known as the Fourier Transform, defined thusly:

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j\,\omega t}\, dt$$

Note that this is quite similar to the formula used for finding the C_n's. Instead of giving us the individual C_n's however, this gives us the formula for the envelope of the C_n's in the f-domain.

If we compute the transform of a single square wave using this formula, we get the following result for the form of the solution:

$$X(f) = \frac{\sin x}{x} \rightarrow \text{sinc } x$$

This is the same result we get if we compute the C_n's for a square wave of $T \rightarrow \infty$ and solve for the formula of the envelope.

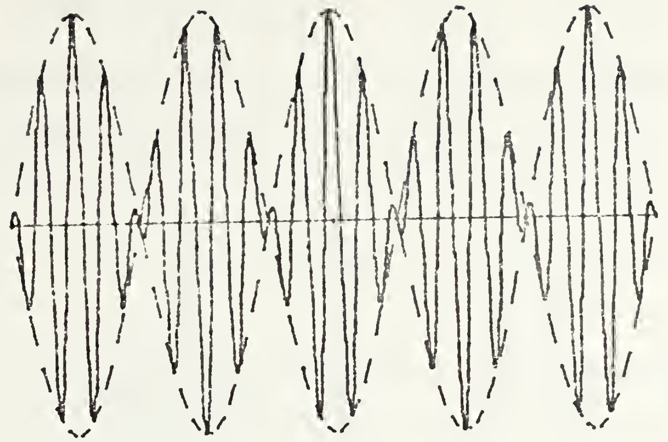
Since we can find the spectrum, or f-domain characteristics of a signal, given its time-domain formula, it is only reasonable that we should be able to do the reverse operation. This turns out to be the case, and the operation is accomplished by using the "inverse transform".

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j\,\omega t} df$$

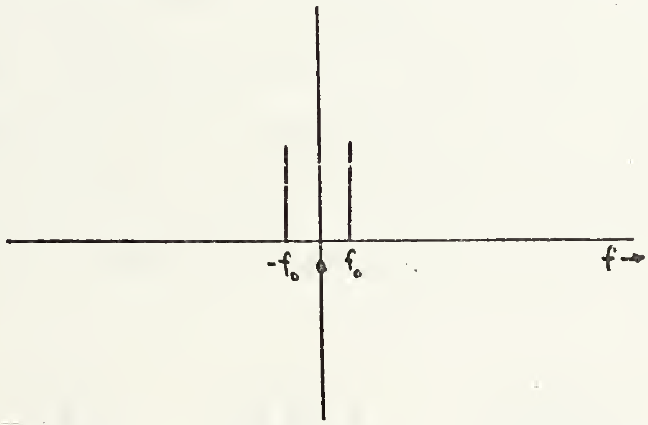
By substitution in the f-domain formula of a signal and then integrating, one can get the time domain characteristics of the signal. The two transform formulas, forward and inverse, are called a "transform pair". One finds the f-domain characteristics given the t-domain formula, and the other finds the t-domain characteristics given the f-domain formula. Later we will see that these are powerful tools used in signal processing.

For the moment, let us return to the cosine wave which we looked at earlier and see what happens if we introduce modulation. That is, superimpose it on a carrier frequency cosine wave similar to AM radio transmissions.

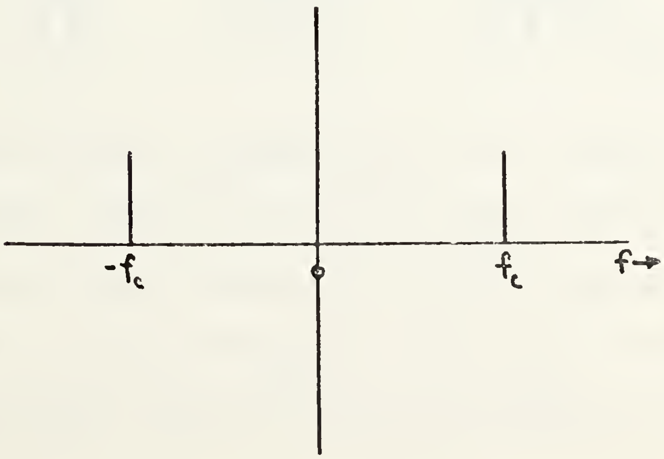
A modulated cosine wave looks as pictured on the following page in the time domain:



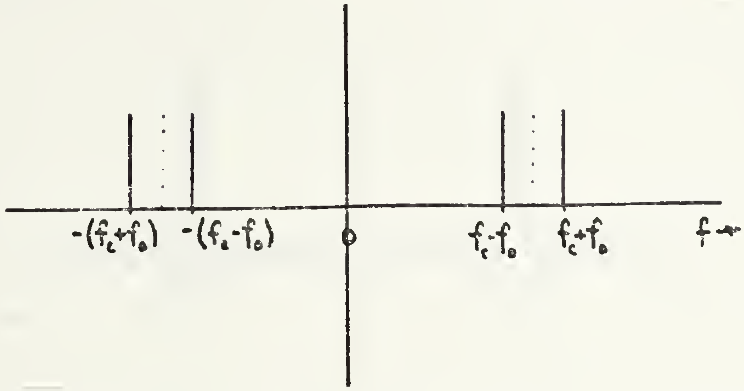
where the envelope varies as the cosine wave we are looking at and the individual peaks are at the carrier frequency. What does this signal look like in the f-domain? We know what the f-domain plot of our cosine wave looks like.



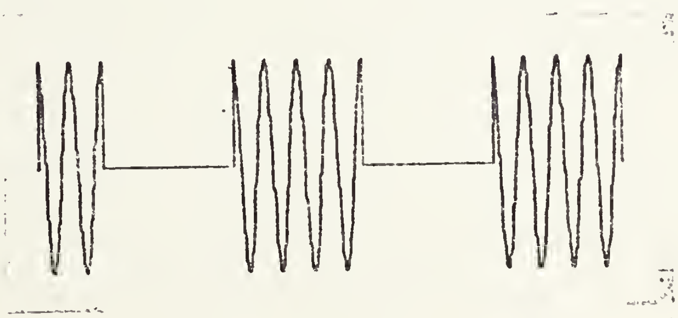
The carrier is just a cosine wave of a different frequency so its plot looks like this.



Note that both f-domain plots were centered about f = 0. Now, if we put the two together, i.e., modulate the carrier with our cosine wave, we get the following:

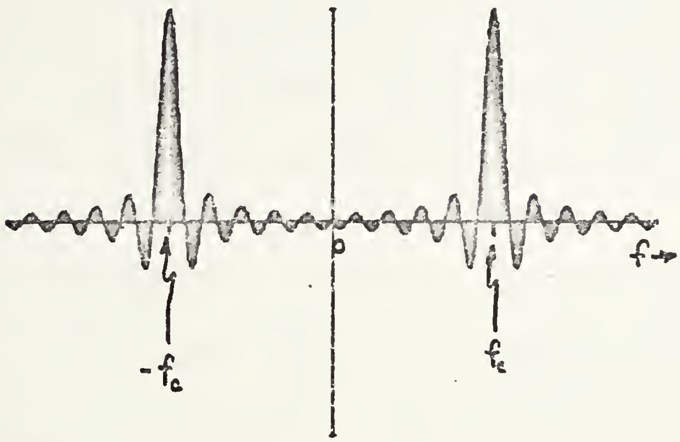


We find that the resulting signal is of the form Cosine $(f_c \pm f)$. The f-domain plot is the cosine wave plot centered about the carrier frequency values instead of about f = 0. Thus, modulation in the time domain is equivalent to a translation of the spectrum in the f-domain from centered about the f= 0 axis to centered about the carrier frequency. Let us look at a modulated square wave pulse train.



We know that the f-domain plot of a square wave is a sinc function. We have just seen that modulation of a signal translates the f-domain plot of the signal and centers it about the carrier frequency. Thus the f-domain plot of a modulated square pulse train is as shown on the following page.

where the envelope takes the shape of the square wave spectrum but is centered about the carrier frequency instead of about $f = 0$.



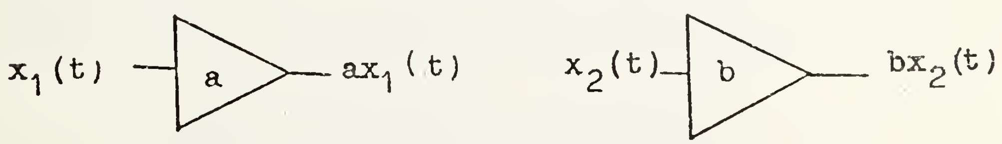
II. FOURIER TRANSFORM PROPERTIES

The properties of Fourier Transforms are stated most succinctly in the accompanying Fourier Transform Theorems. Knowledge of the theorems is important in the interpretation of spectra as the theorems express the relationship between time-domain and frequency-domain operations.

In the introduction to Fourier Transforms it was shown that the frequency spectrum, X(f), of a signal, x(t), was the Fourier Transform of the signal. That is, $X(f) = \mathcal{F}[x(t)]$ and inversely, $x(t)=\mathcal{F}^{-1}[X(f)]$. These relations are called "transform pairs" and are more conveniently denoted by $x(t) \leftrightarrow X(f)$. The Fourier Transform Theorems describe the properties of various Fourier Transform pairs.

A. LINEARITY (OR SUPERPOSITION) THEOREM

> If a signal, $x_1(t)$, is multiplied by a constant, a, (as would happen if an amplifier with gain=a were placed in a circuit) and a signal, $x_2(t)$, is multiplied a constant, b, then their respective line spectra will also be multiplied by the constant a or b.



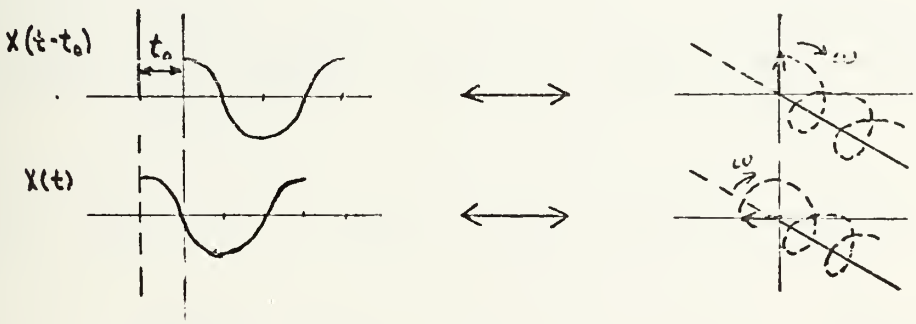
$$ax_1(t) + bx_2(t) \leftrightarrow aX_1(f) + bX_2(f)$$

Linear combinations in the time-domain become linear combinations in the frequency-domain.

B. TIME DELAY THEOREM

> Given the transform pair $x(t) \leftrightarrow X(f)$ if the signal, $x(t)$, is delayed by t_o seconds, as happens when the signal passes through a capacitive or inductive element, to become a new signal, $x(t-t_o)$, then the spectrum is modified by a frequency dependent phase shift to become $X(f)e^{-j\omega t_o}$.

$$x(t-t_o) \leftrightarrow X(f)e^{-j\omega t_o}$$



The translation of a signal in time changes the phase of the spectrum but does not alter the complex amplitude.

To illustrate the Linearity and Time Delay Theorems, consider a signal which is a linear combination of delayed signals:

$$y(t) = 2x(t-t_o) - 4x(t-3t_o)$$

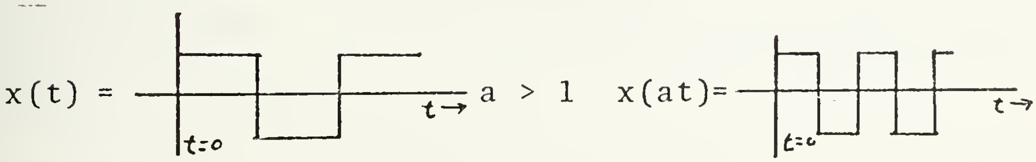
Its spectrum is easily written as:

$$Y(f) = 2X(f)e^{-j\omega t_o} - 4X(f)e^{-j\omega 3t_o}$$

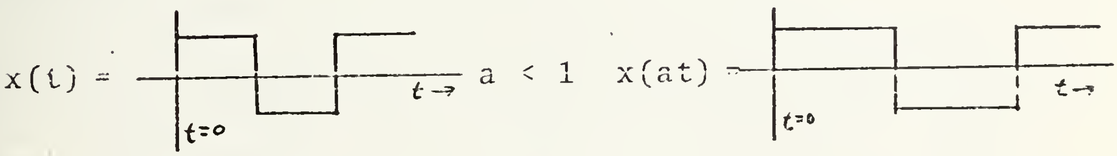
C. SCALE CHANGE THEOREM

It has been shown that a translation of the time origin may be accomplished using the Time Delay Theorem. The time axis may also be expanded, compressed, or reversed by an operation known as "scale change". If a signal, $x(t)$, becomes a new signal, $x(at)$, then:

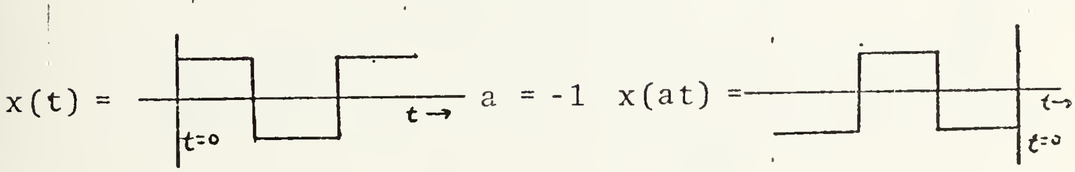
(1) $x(at)$ is compressed if <u>a</u> is positive, greater than one;



(2) $x(at)$ is expanded if <u>a</u> is positive, less than one;



(3) $x(at)$ is reversed in time if <u>a</u> is negative.



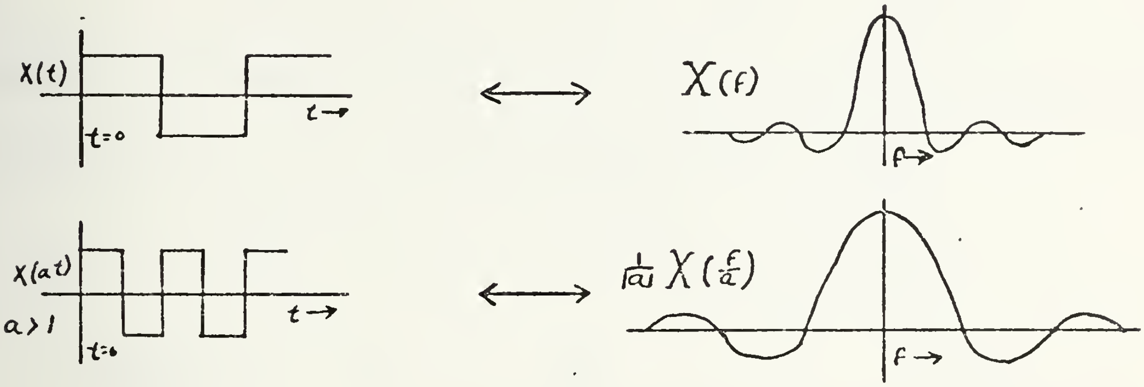
In the Fourier Transform pair,

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right),$$

$|a|$ means the absolute value of a, that is, the numerical value only is used, the sign is dropped.

The scale change theorem expresses the property of reciprocal spreading. If the signal is compressed in time by the

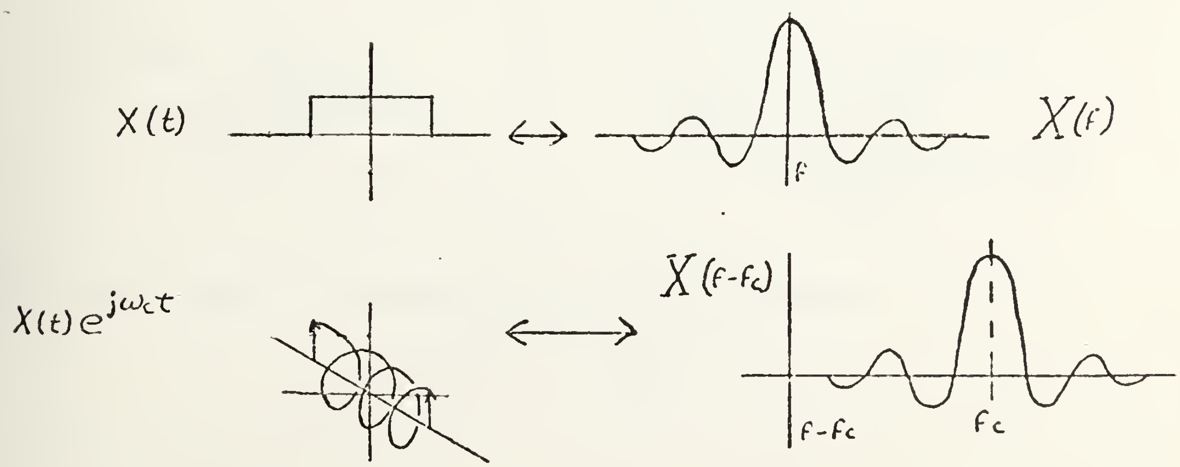
factor a, its spectrum is expanded in frequency by 1/a. If the signal is reversed in time (a = -1) its spectrum is $X(\frac{f}{-1}) = X(-f)$, which indicates a reversal in phase.



The expansion or compression in the time axis of a signal occurs most often in the playback of recorded signals.

D. FREQUENCY TRANSLATION (MODULATION) THEOREM

> If a signal, x(t), is multiplied by $e^{j\omega_c t}$, its spectrum, X(f), will be translated in frequency by $\pm f_c$. Mathematically stated:
>
> $$x(t)e^{j\omega_c t} \leftrightarrow X(f-f_c).$$

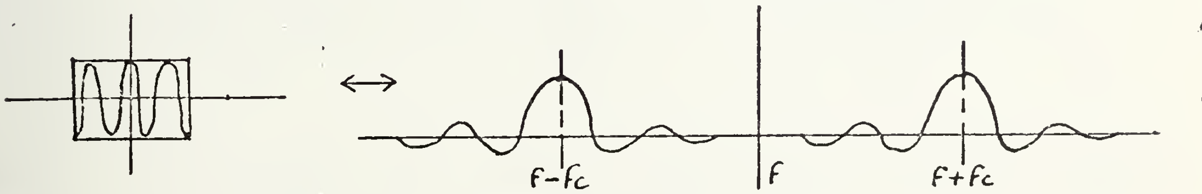


It is more common to multiply a time signal, x(t), by the real part of $e^{j\omega_c t}$, i.e., by cos $\omega_c t$. From previous work with Euler's theorem, it is recalled that

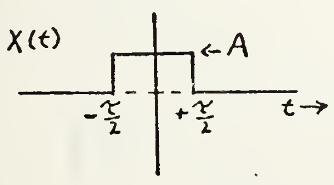
$$x(t)\cos\omega_c t = 1/2x(t)(e^{j\omega_c t} + e^{-j\omega_c t}).$$

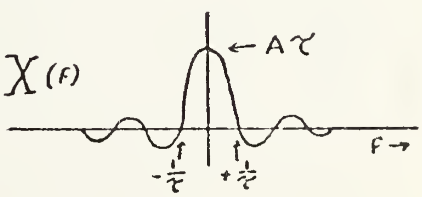
Taking this result and applying it to the Frequency Translation Theorem yields the Frequency Modulation Theorem:

$$x(t)\cos\omega_c t \leftrightarrow 1/2X(f-f_c) + 1/2X(f+f_c)$$



Or, if a signal x(t) is multiplied by cos $\omega_c t$, its spectrum X(f) is shifted up and down in frequency by an amount f_c. This is the same result noticed earlier when an RF pulse train was formed by multiplying a rectangular pulse train by cos $\omega_c t$. As noted then, and confirmed by the modulation theorem, time domain multiplication becomes translation in the frequency domain. As an example, a rectangular pulse of amplitude A, period t, centered at t = 0 and duration τ is $x(t) = A\Pi(t/\tau)$.



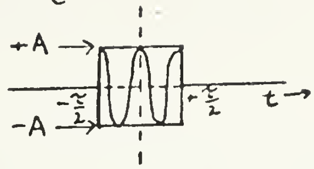


The spectrum is recognized as a SINC function, or

$$X(f) = A\ \mathrm{SINC}\ f\tau.$$

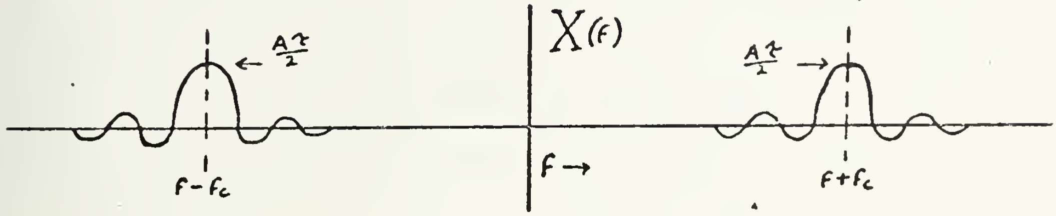
Now, multiply that single pulse by cos $\omega_c t$:

$$x(t) = A\Pi(t/\tau)\cos\omega_c t.$$



By the modulation theorem, the spectrum is:

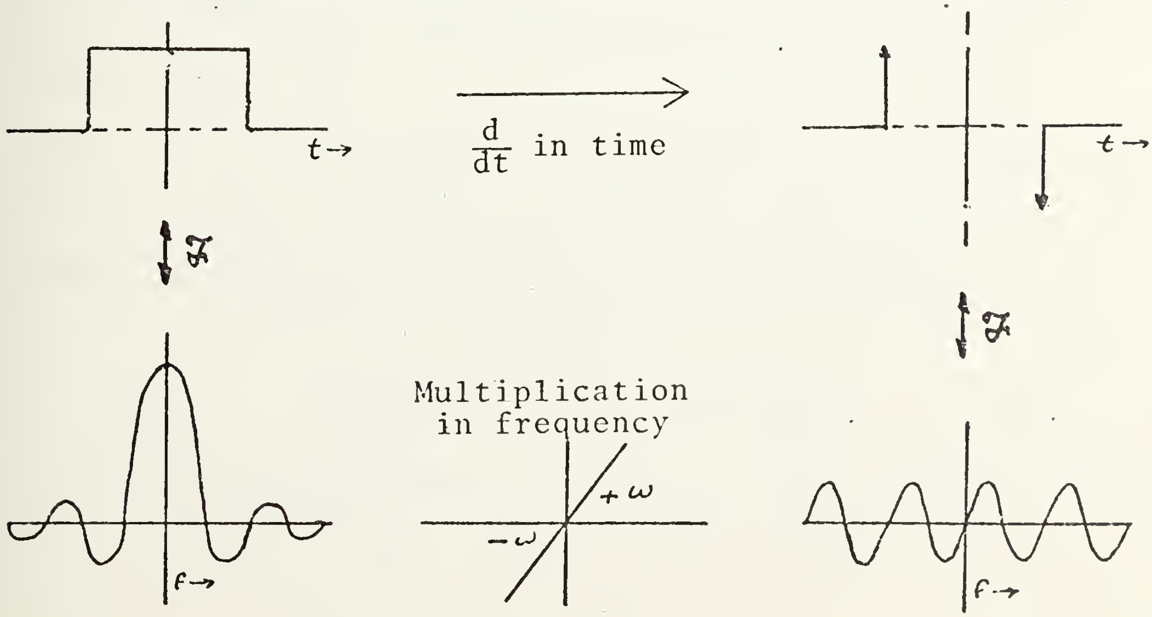
$$X(f) = 1/2\ A\ \mathrm{SINC}\ (f-f_c)\tau + 1/2\ A\ \mathrm{SINC}\ (f+f_c)\tau.$$



This is the result that is used extensively in single-sideband and double-sideband HF communications.

E. TIME DERIVATIVE THEOREM

> If the derivative, $\frac{d}{dt}$, is taken of a signal x(t), the result is multiplication in the spectrum by $j\omega$:
>
> $$\frac{d}{dt}x(t) \longleftrightarrow j\omega X(f).$$

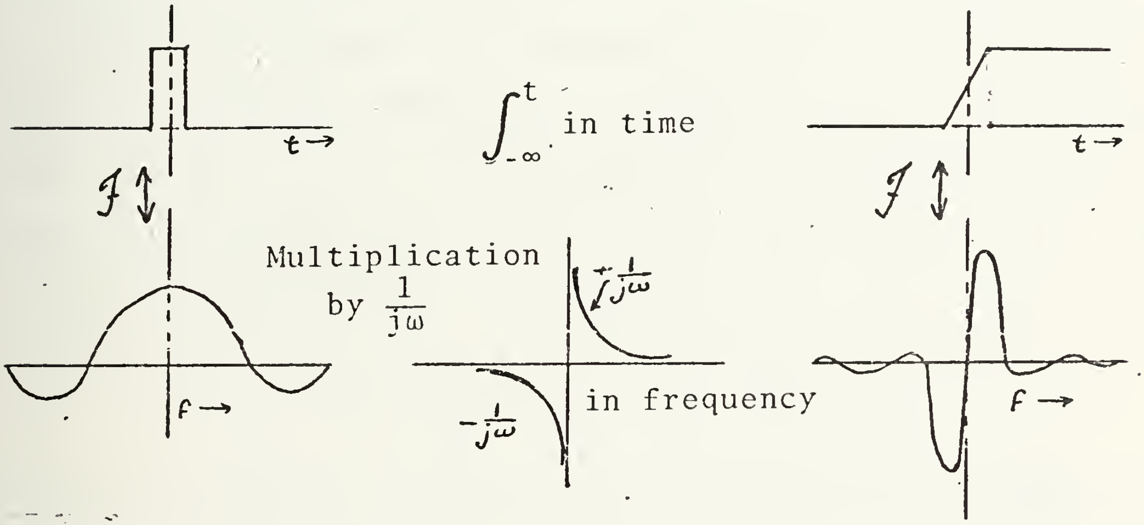


Therefore, differentiation enhances the high frequency components in the spectrum.

F. TIME INTEGRATION THEOREM

Taking the integral $\int_{-\infty}^{t}$ of a signal $x(t')$ is equivalent to multiplying by $\frac{1}{j\omega}$ (dividing by $j\omega$) in the frequency domain:

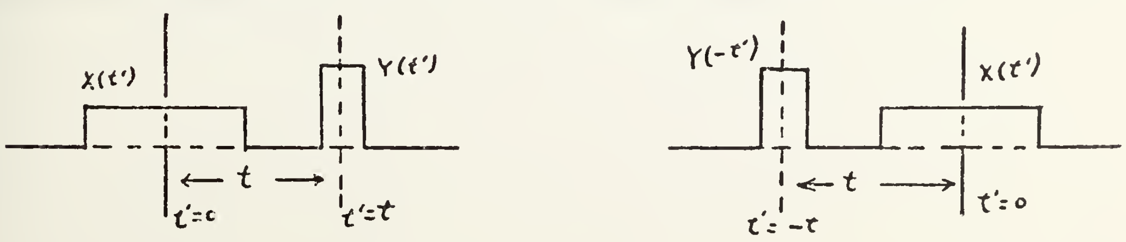
$$\int_{-\infty}^{t} x(t')\, dt' \longleftrightarrow \frac{1}{j\omega}\ X(f).$$



Integration, then, suppresses the high frequency components but the low frequency components are unaffected.

G. Convolution

Take two signals in the time domain t', for example, $x(t')$ and $y(t')$. Now "flip" the $y(t')$ signal to the other side of the axis and it becomes $y(-t')$ as illustrated below.



In order to make a comparison between the two signals, multiply them together at every point in time t' from $-\infty$ to $+\infty$. Mathematically, this can be expressed as

$$z = \int_{-\infty}^{\infty} x(t')\ y(-t')dt' .$$

In the above case, anywhere in the t' domain where $x(t')$ exists, $y(-t') = 0$, and where $y(-t')$ exists, $x(t') = 0$. Therefore, their product is zero.

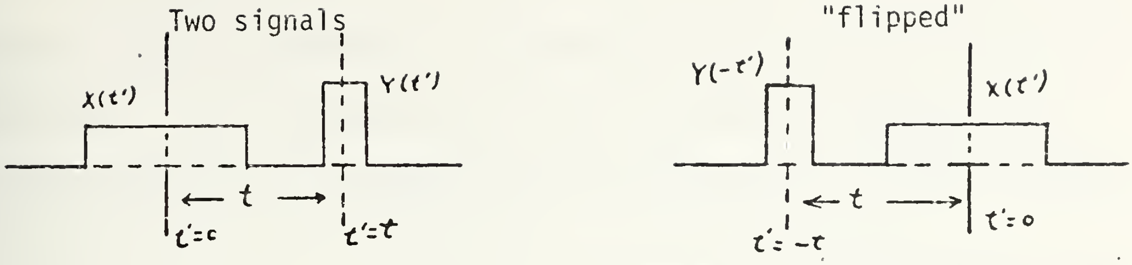
Since the signals are separated by time t, if a time delay of t is introduced making $y(-t')$ become $y(t-t')$, the signals will overlap. If t, the time delay, is varied from $-\infty$ to $+\infty$, the $y(t-t')$ signal will "slip" across the signal $x(t')$. The result is expressed as a function of time, t:

$$z(t) = \int_{-\infty}^{+\infty} x(t')\ y(t-t')dt' .$$

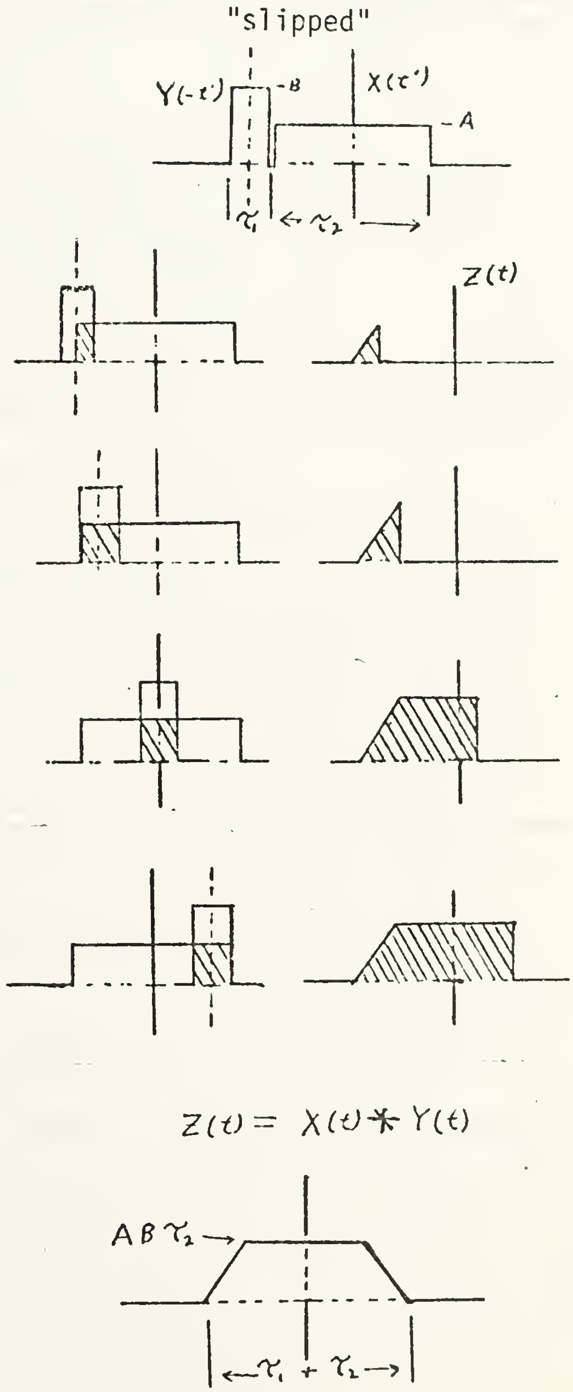
The above integral is known as the "convolution integral" and it expresses convolution mathematically, i.e., the area of the product of the two signals. A more convenient notation for convolution is:

$$z(t) = x(t) * y(t) .$$

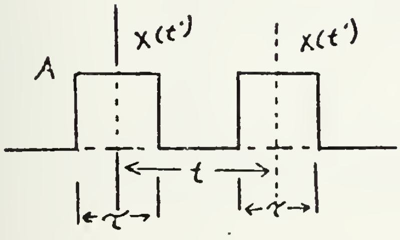
A graphical interpretation of convolution, as shown on the following page, may help to see what happens to the signals.



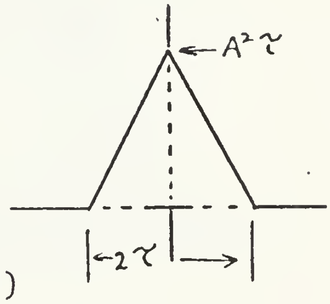
The shaded area on the time plot of both signals represents the area of their "common product" for a particular value of time delay, t. The shaded area on the z(t) plot represents the summation of the area of their "common product" as the "flipped" signal, y(-t), is "slipped" across x(t).



The convolution, i.e., the "flip" and "slip", of two rectangular pulses produces a trapezoidal pulse with a base of length $\tau_1 + \tau_2$. Convolution of two identical rectangular pulses produces a triangular pulse with base = 2τ and height = $A^2\tau$.



$z(t) = x(t) * Y(t)$
in this instance,
$z(t) = x(t) * x(t)$



Now that convolution in the time domain has been explained, how does it relate to the frequency domain? There are two convolution theorems which relate the two domains.

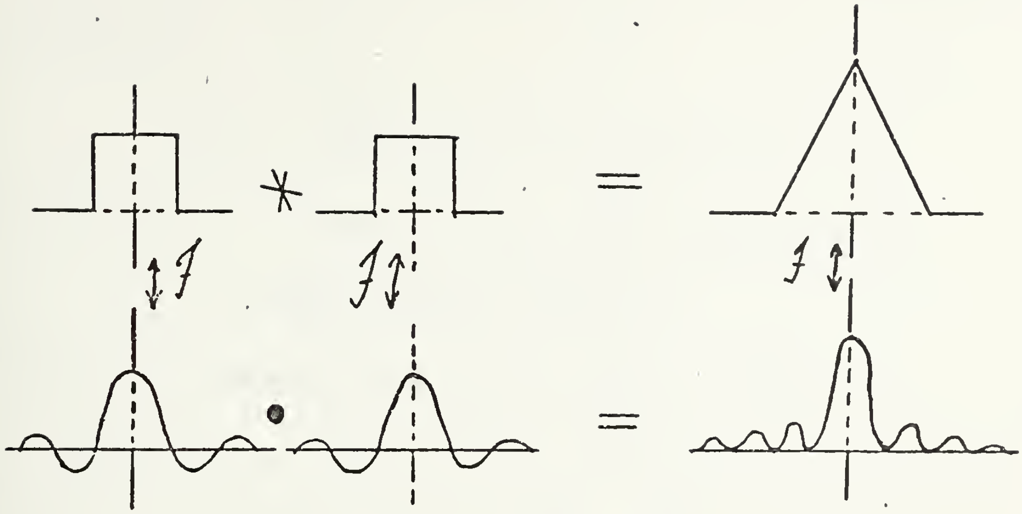
$$x(t) * y(t) \Leftrightarrow X(f) \cdot Y(f)$$

Convolution in the time domain transposes to multiplication in the frequency domain.

$$x(t) \cdot y(t) \Leftrightarrow X(f) * Y(f)$$

Multiplication in the time domain transposes to convolution in the frequency domain.

For example, look at the now familiar square pulse shown on the following page.



H. Correlation

In statistics, when it is desired to know how closely one distribution resembles another, the two are overlayed and the areas they have in common are multiplied. This "common product" represents the degree of similarity, or how well one distribution correlates to the other. In signal processing it is often necessary to compare one signal with the same signal which occurs later in time or with another signal. Signals vary in time in contrast to distributions which are constant in time. Therefore, in order to completely overlay one signal on another, displace the signal in time by τ units, then vary τ to "slip" the displaced signal across the one with which it is desired to correlate. With each different τ used, the "common product" will be a different value. In order to obtain one number to represent the value of correlation, sum the "common product" over all values of delay, τ, and average it by dividing by time T (or multiplying by $\frac{1}{T}$). This

defines the "correlation function" $R(\tau)$ and is described mathematically as:

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_T x(t)\ x(t+\tau)dt\ .$$

This is known as a "time average" and is denoted by

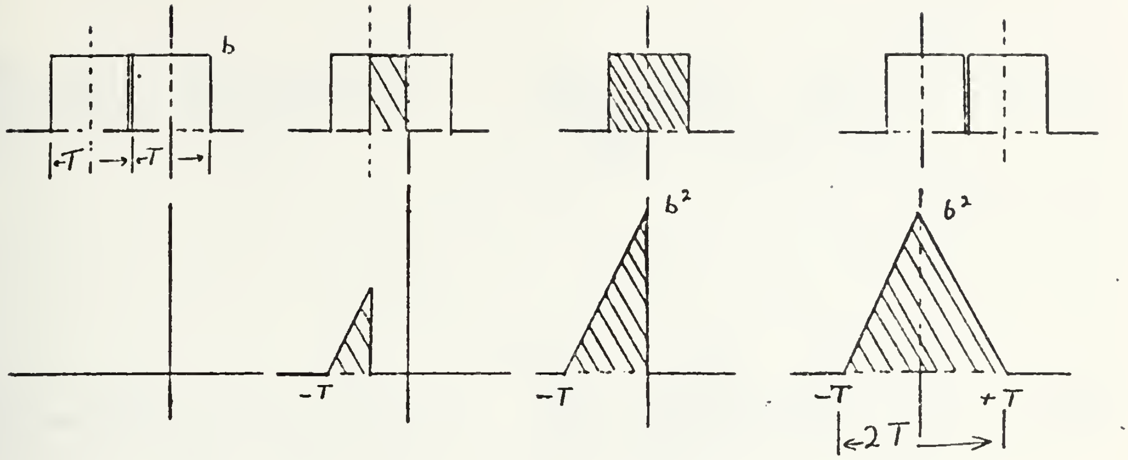
$$R(\tau) = \langle x(t)\ x(t+\tau)\rangle,$$

where the $\langle\ \rangle$ (brackets) indicate $\lim_{T\to\infty} \frac{1}{T} \int_T (x(t)\ x(t+\tau)dt$. If the signal is periodic in T_0 it need only be averaged over one period, T_0, instead of all time. The correlation function then becomes:

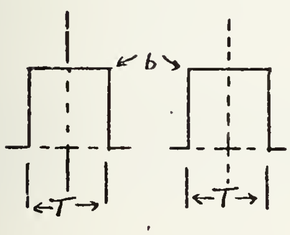
$$R(\tau) = \frac{1}{T_0} \int_{T_0} x(t)\ x(t+\tau)dt\ .$$

$R(\tau)$ is more precisely referred to as the autocorrelation function, that is, the correlation of one signal with itself. It may be desirable, as mentioned earlier, to correlate one signal, $x(t)$, with another signal, $y(t)$, displaced τ units in time to become $y(t+\tau)$. This results in the cross correlation function, $R_{xy}(\tau) = \langle x(t)\ y(t+\tau)\rangle$. The autocorrelation function is more commonly used. Where "correlation function" is used it is assumed to be the "autocorrelation function".

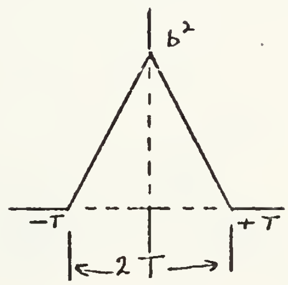
For a graphical representation, the familiar square pulse is used. This, as shown on the following page, represents the result of the summation of the "common product" as the delayed pulse is "slipped" across the other pulse. This initial part of correlation is like the previously described convolution except that the signal is not flipped.



The next step is to take the time average of the summation of the common products. It is seen that, as in convolution, correlation increases the width of the function by $T_1 + T_2$, or, in the case of a square pulse, from T to 2T.

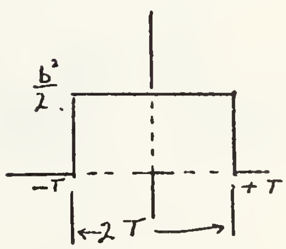


Sum of
common products =



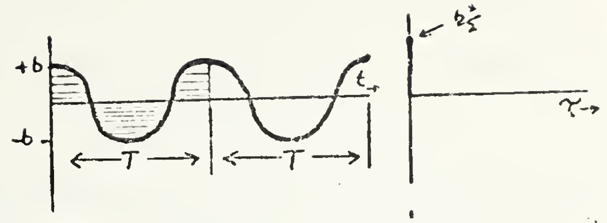
To time average, take the area = 1/2 bh, or $1/2(2T)b^2$, and divide by time, 2T,

$$\frac{1/2(2T)b^2}{2T} = \frac{b^2}{2} \; .$$

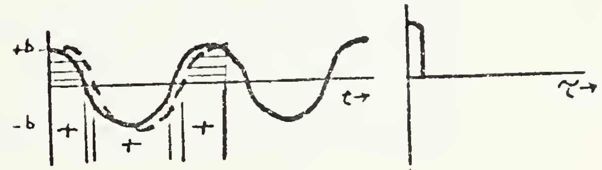


What does the correlation of a sinusoidal signal look like? [L]et us see on the following page.

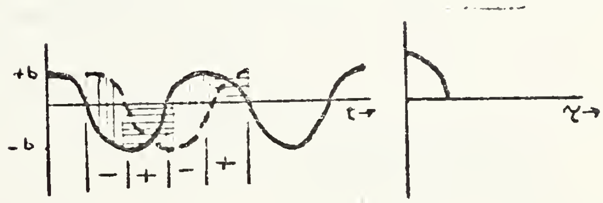
When both signals overlay completely, the value is large and positive.



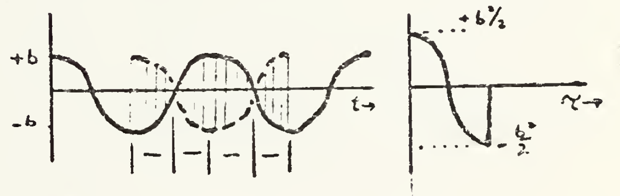
A small shift will give some negative area but the result is still positive.



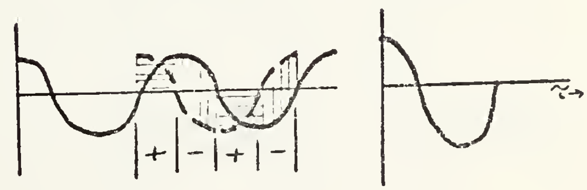
A 90° shift makes the positive and negative areas equal and the resultant value zero.



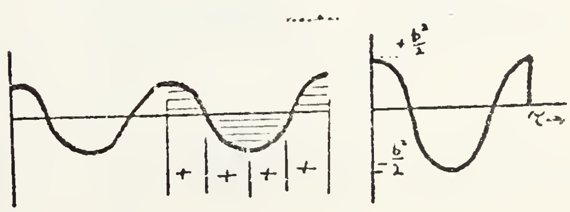
A shift of 180° results in the entire area being negative.



At 270° the positive and negative areas are again equal and the resultant value is zero.

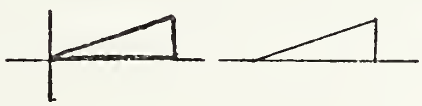


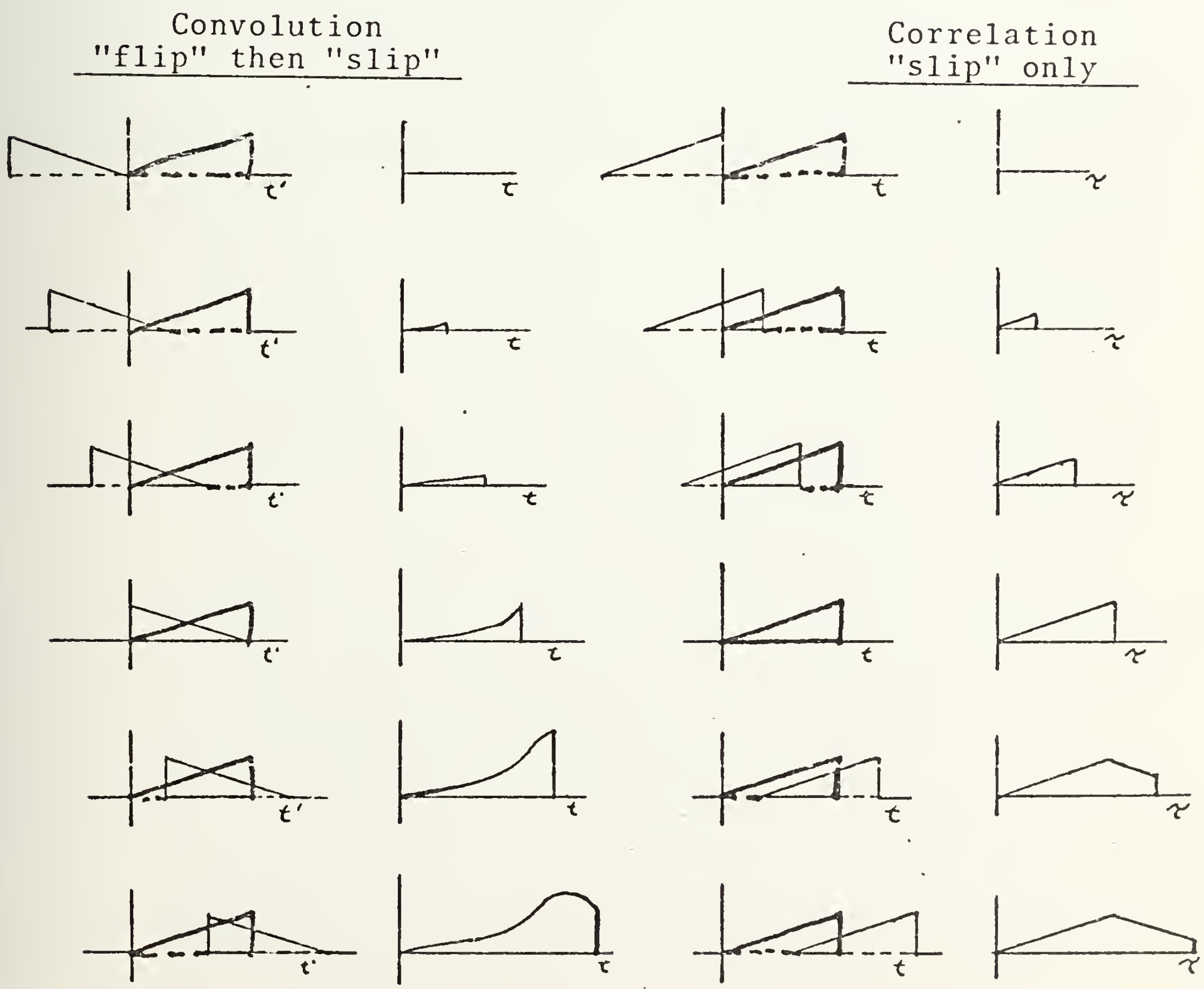
At 360°, a complete one cycle shift, the signal is back in phase and the result is a maximum positive value.



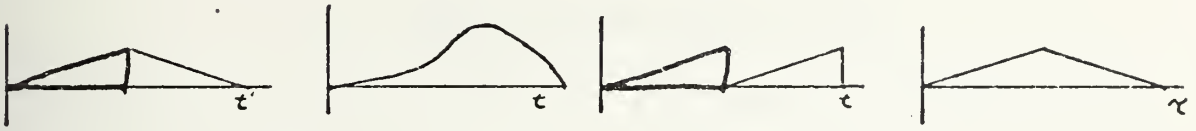
The correlation function of a sinusoidal signal is another sinusoid, and, if the time average is taken, has an amplitude of $\frac{b^2}{2}$.

Up to now there have appeared many similarities between convolution and correlation. The graphical interpretation has also shown this similarity but much of that is due to the choice of symmetrical signals for illustration. Now, look at the side by side convolution and correlation of a saw-tooth signal.





The final results are indicated in the next diagram.



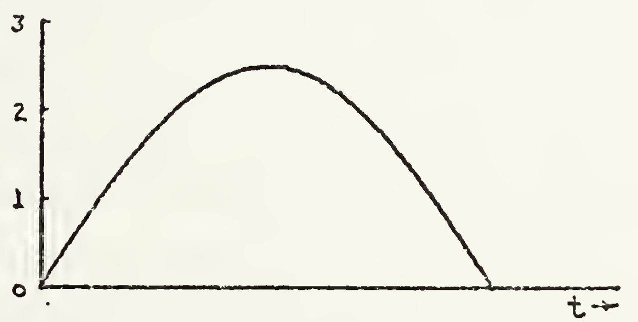
When "time Averaged" becomes:

So it is seen that if an asymetrical signal is used, the final result obtained by convolution may be quite different from the final result obtained by correlation.

III. SAMPLING AND QUANTIZATION

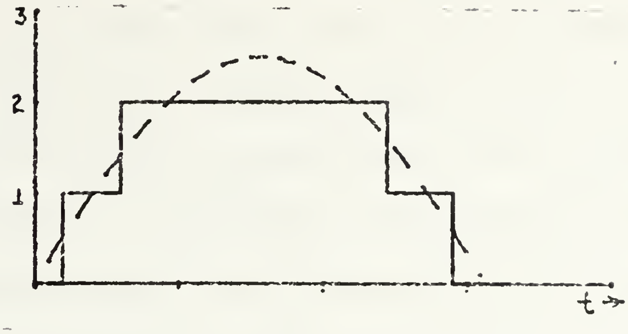
In the real world, most of the signals of interest to an operator of a detection system are of a continuous, or analog, nature. On the other hand, most of the more sophisticated processors in use today are digital in operation, and thus can not use analog inputs directly. Some means of digitizing the analog signals must be used.

To digitize a signal, one must describe the value of the signal at any given time by a digital number. This requires a measurement and conversion of the signal value to digital units. The question of how accurately we must measure the value of the signal in converting it to digital form must be considered. Take the following signal, for example:

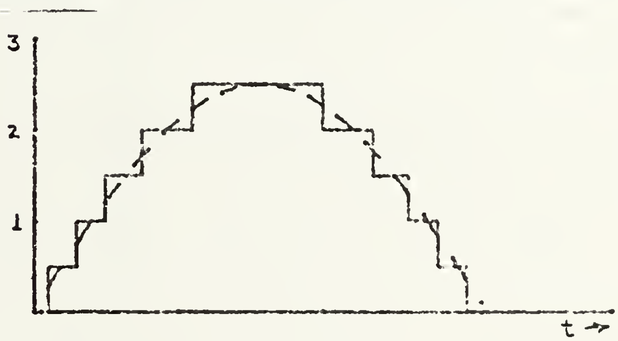


This signal has a maximum value of 2.5 and a minimum value of 0. Since we must convert the continuous values of the signal to a set of discrete values, we must then decide how many discrete values we need in the range from 0 to 3 to describe the signal adequately. If we decide to use four levels, that is, digitize to the nearest integer value, the signal

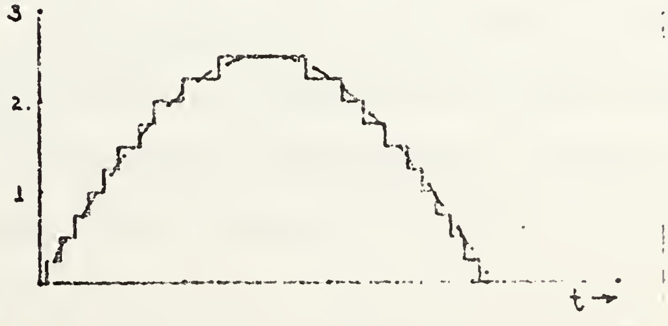
will have the following appearance.



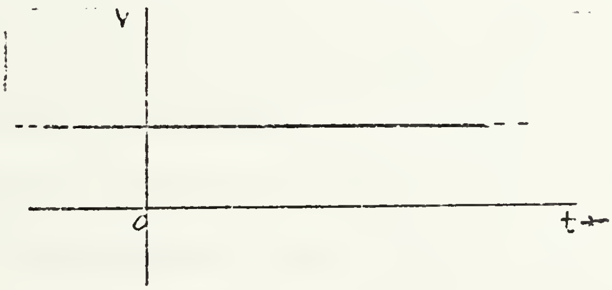
If we double the number of values and digitize to the nearest half integer value, the digitized signal looks like this.



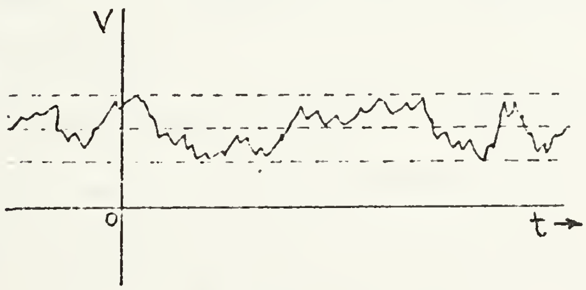
Note that as the number of values used in digitizing the signal increases, the digitized curve more and more closely approximates the original signal. The values used in digitizing the signal are known as quantization levels. In the case of a noise-free signal, one can digitize the signal to any degree of accuracy by increasing the number of quantization levels.



The case of a signal with noise is different, however. In this case, there is a random fluctuation of the signal value because of noise even when the value represented by the signal itself is unchanging. The range of these noise fluctuations determines how accurately we can measure the value represented by the signal even if we have a perfect measuring device. To illustrate this, consider first a signal without noise and then one with noise.



WITHOUT NOISE



WITH NOISE

We observe that in the noisy signal, the value of the signal fluctuates about some mean, or average, value within some limits. If the noise is truly random in nature, the mean value about which it fluctuates is then the actual value of the signal being represented. At any given instant, any value of the signal between the upper and lower limits

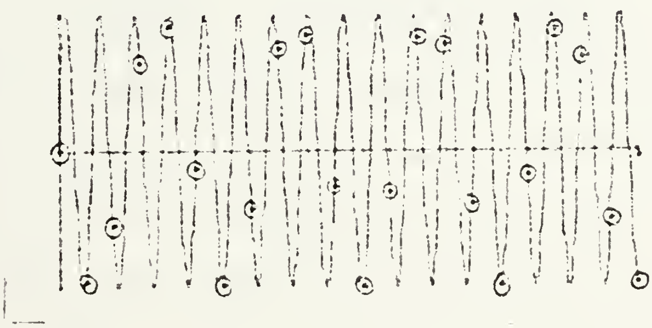
of the noise fluctuation could stand for the actual value being represented. The accuracy with which this signal can be measured is then limited to the range of fluctuation due to noise. For example, suppose the signal of interest is a constant 3.0 volts, but because of noise the signal fluctuates between 3.5 and 2.5 volts. The most accuracy with which we can measure this signal is to the nearest integer volt.

Once we determine how many quantization levels are necessary, we mjst then determine how often we need to sample the signal in order to reconstruct it accrately. Obviously, if the signal never changes value, but remains constant, only one sample is needed to reconstruct the signal for all times. If, however, the signal is not constant, we face a real problem.

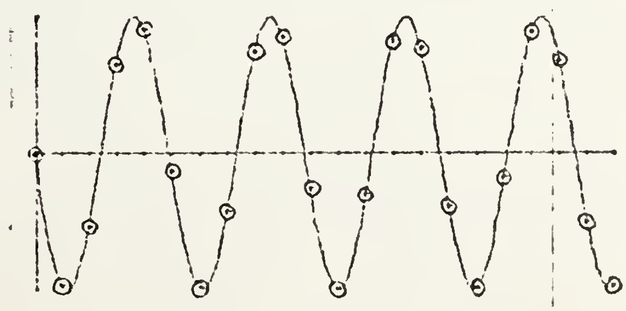
How often do we have to sample a given signal? Well, we know that one sample is not enough if the signal is not constant, and we also know that if the signal is sampled continuously (i.e., in an analog fashion) we can reproduce it exactly. To sample a signal at extremely short intervals is very costly and difficult to accomplish. What we are looking for is a compromise, a sampling rate high enough that the signal can be reproduced from the samples but not any higher than necessary in order to keep cost and equipment complexity down.

We can see intuitively that there must be some relationship between the rate of change of the signal and the sampling rate

necessary to be able to reproduce it. If the signal changes very slowly, we need sample it only infrequently. If it changes rapidly, however, we must sample it frequently enough that it does not have time to make several changes in the interval between samples or we lose these changes and are thus unable to reproduce the signal. If the sample intervals are too long, the signal is said to be "undersampled". This condition, in addition to causing us to lose some of the information in the signal, can also cause other complications. Perhaps you have noticed that in some of the old cowboy movies the stagecoach wheels appear to turn backwards at times. This is a case of undersampling which illustrates one of the other effects called "aliasing". To explain this, let us look at a sine wave.

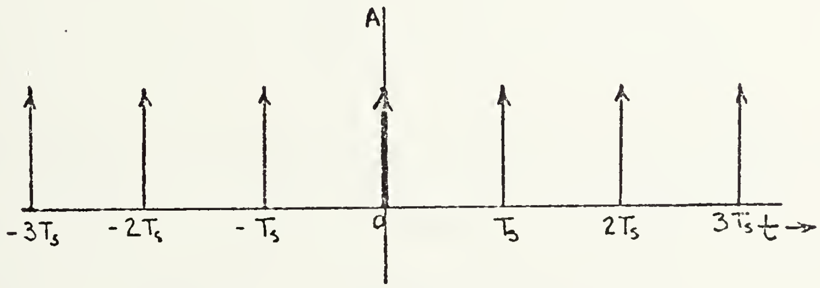


If we sample this at a frequency less than that necessary for reproduction of the signal, we can get a series of samples which give the impression of a sine wave of lower frequency.



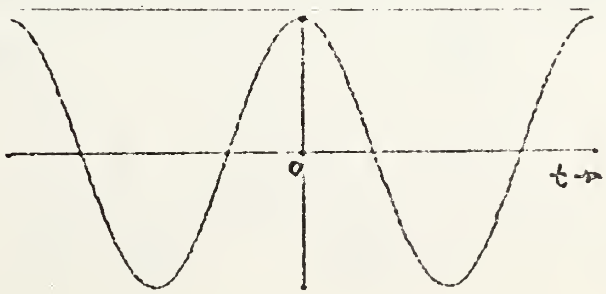
This is a problem which must be addressed when analyzing unknown signals for frequency components since aliasing can give spurious results if the analyzer sampling rate is not high enough. This will be discussed further in a minute. In the meantime, let us look at the sampling process.

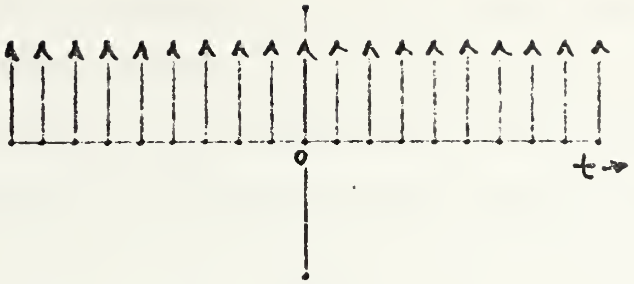
In the time domain, we can visualize the sampling process ideally as the multiplication of the signal by a function known as the "ideal sampling wave", which is identically equal to zero except at the sample times [T_s, $2T_s$, etc.].

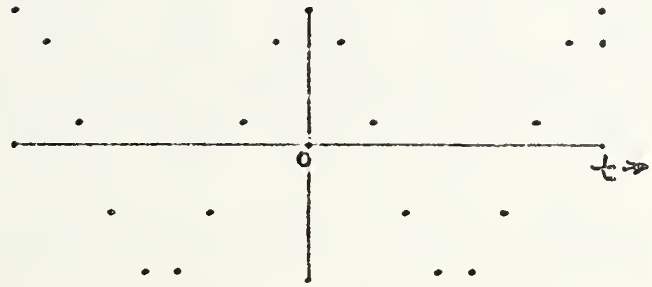


"IDEAL SAMPLING WAVE"

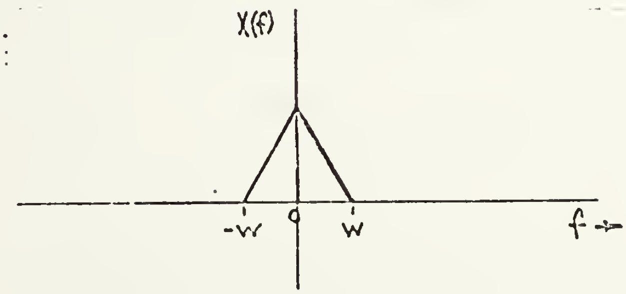
The duration of these spikes, known as "delta functions", is infinitesimal, approaching zero, while their amplitude approaches infinity, and they are defined such that their area ($0 \times \infty$) is equal to 1. When the signal of interest is multiplied by the ideal sampling wave, we get the product of their areas at each point T. Since the area of each delta function is 1, and its width is 0, the product of areas is equal to the value of the signal at that time. Thus the products of the two functions appear as below.



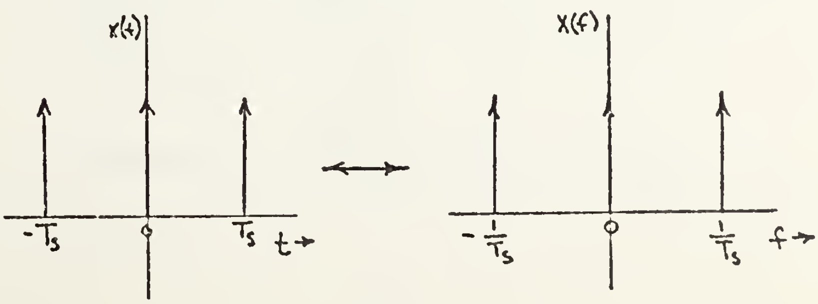




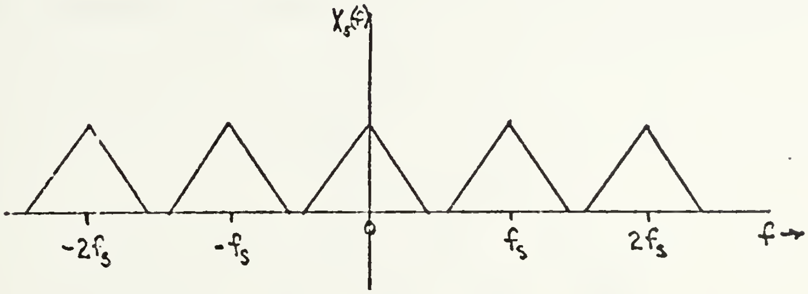
Let us look at the frequency domain to see more clearly what happens when a signal is sampled. Assume that the signal to be sampled is strictly "bandlimited", that is, it has no frequency components outside of some limit.



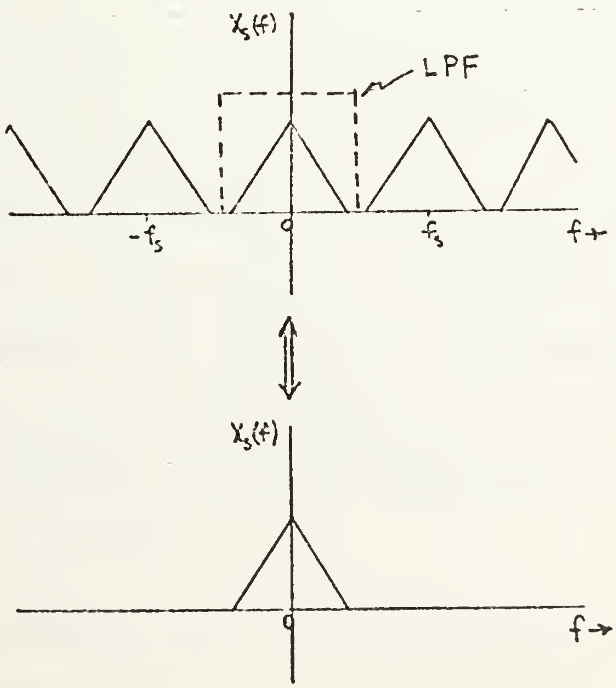
2W is the bandwidth of this signal. Although the mathematics is rather involved, it can now be shown that the ideal sampling wave spectrum appears as in the next diagram.



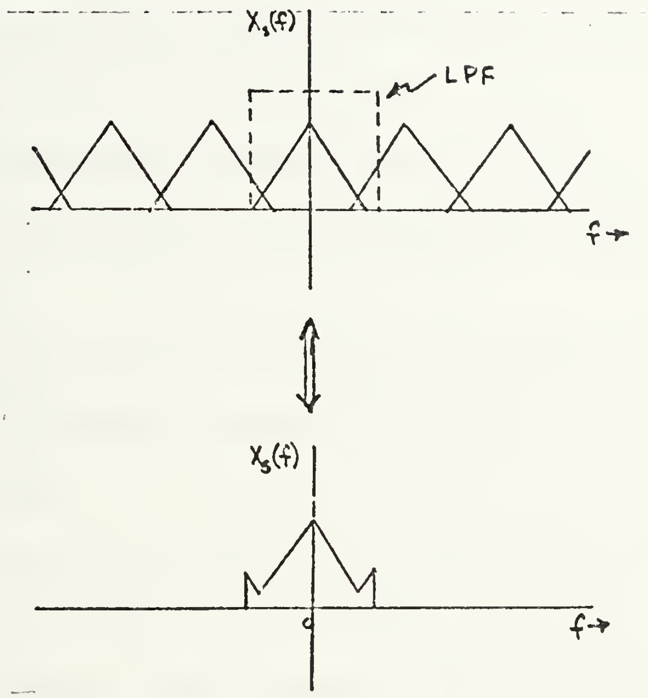
We know from the modulation theorem that modulation of a function causes its spectrum to be shifted and centered about the modulation frequency and otherwise does not change it. Thus, the spectrum of the sampled signal appears as follows.



From this we can see that for strictly bandlimited signals, if $f_s \geq 2W$, there is no overlapping of the spectrum components and so the spectrum of the original signal is exactly reproduced. If the sampled signal is then passed through a low-pass filter which has the property of allowing only those frequencies below its cutoff to pass, we can exactly recover the original signal.



It follows from the above that the sampling frequency must be $\geq 2W$, i.e., greater than, or equal to, two times the highest frequency component of the signal if we are able to reconstruct the signal from the samples. If it is not, the spectrum of the sampled signal has overlaps which introduce errors into the reconstruction of the signal.



These overlaps are the cause of "aliasing" and other inconsistencies in the reconstructed signal. This result is formalized in the Sampling Theorem, or theorem of uniform sampling:

> If a signal contains no frequency components for $|f| \geq W$, it is completely described by instantaneous sample values uniformly spaced in time with period $T_s \leq 1/2W$.

This rate of sampling, $f_s = 2W$, is known as the Nyquist Rate.

It is the absolute minimum sampling rate from which a signal can be reconstructed under ideal conditions.

Until now we have considered only ideal sampling and reconstruction. In practice, the ideal is rarely, if ever, realized. Practical sampling differs from ideal sampling in three obvious respects:

(1) Practical sampling waves are not composed of delta functions but have a finite width.

(2) Practical filters are not ideal.

(3) Real world signals normally sampled are not strictly bandlimited.

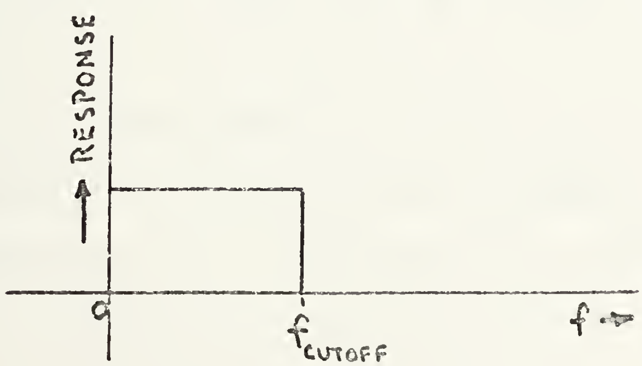
In practice these differences are of small enough significance that for all normally encountered cases only the last is of real importance.

Since most real signals are not strictly bandlimited, there will be some overlap in the sampled spectrum. If the spectral components outside some nominal value W are negligible, however, the signal can still be adequately described for most purposes by samples spaced $T_s \leq 1/2W$.

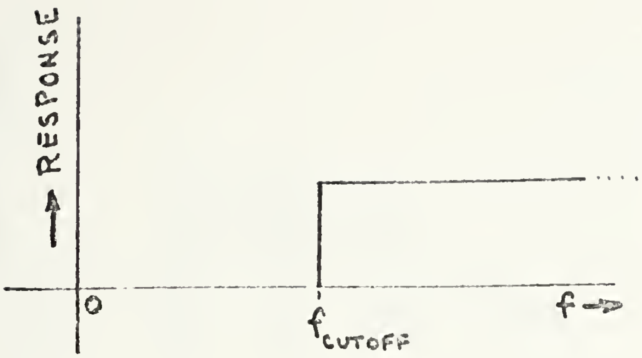
IV. FILTERS AND BANDWIDTH REQUIREMENTS

The ocean is an extremely noisy environment in which to search for submarines. Noise from waves, wind, breaking surf, shipping, sonic mammals, fish and crustaceans, seismic activity, rain, etc., is present to some degree at all times. How do we go about sorting out these different sources and detecting submarines? One way is to use filters to eliminate as much of the noise as possible.

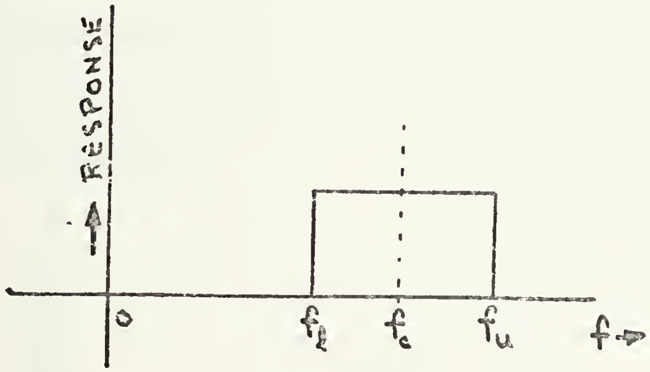
In general, there are three classes of filters relative to spectral response: low-pass, high-pass, and band-pass filters. Ideal low-pass filters allow frequencies below their cutoff frequency to pass, and screen out all above cutoff. Their spectral response is shown in the following diagram.



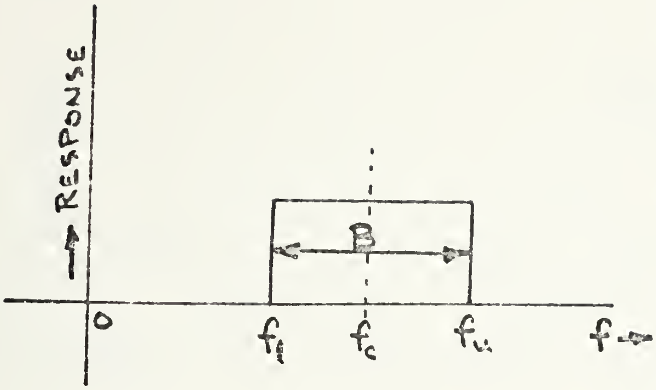
High-pass filters allow all frequencies above their cutoff to pass and screen out all below.



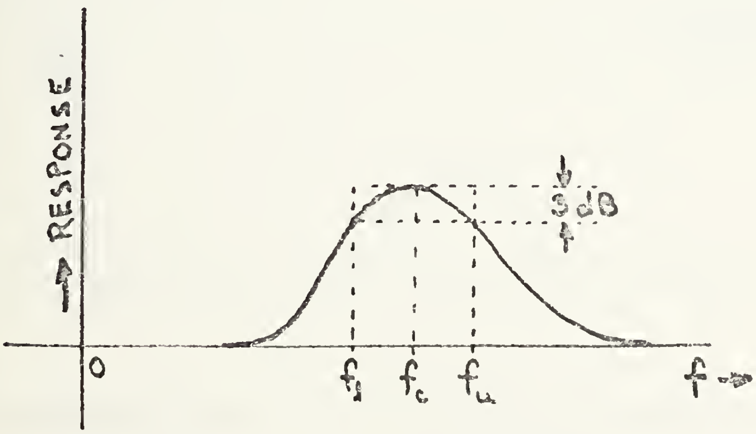
Band-pass filters allow all frequencies between their lower and upper cutoff frequencies to pass and screen out all others.



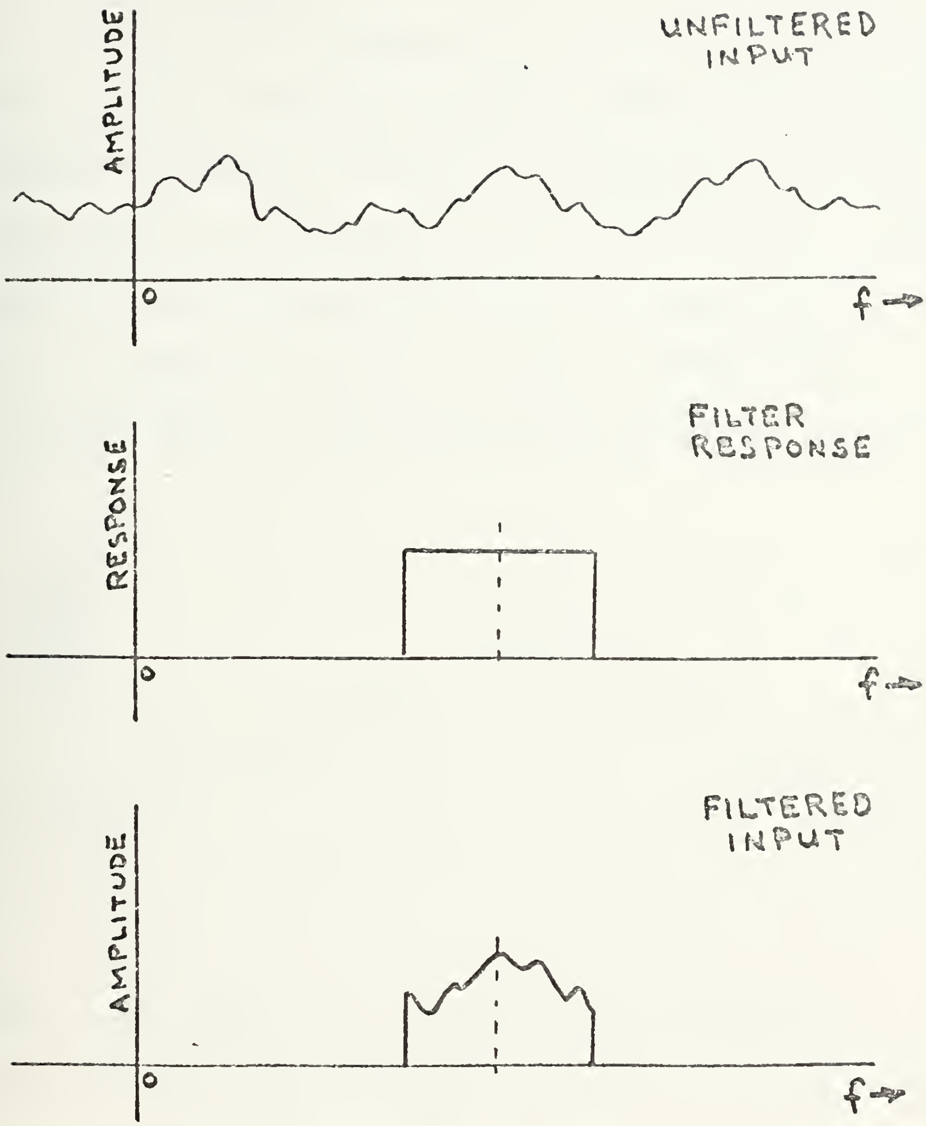
Filters are characterized by their upper and lower cutoff frequencies, or their center frequency and band-pass (or bandwidth), B. The bandwidth is defined as the passband width measured in positive frequency only, that is, the difference $f_u - f_1 = B$.



Of course, no real filters have the sharp cutoff characteristics of the ideal filter, so it is customary to define the bandwidth of a real filter as the bandwidth between the points where the response has dropped 3 dB from the maximum. These points are known as the "3 dB down points".



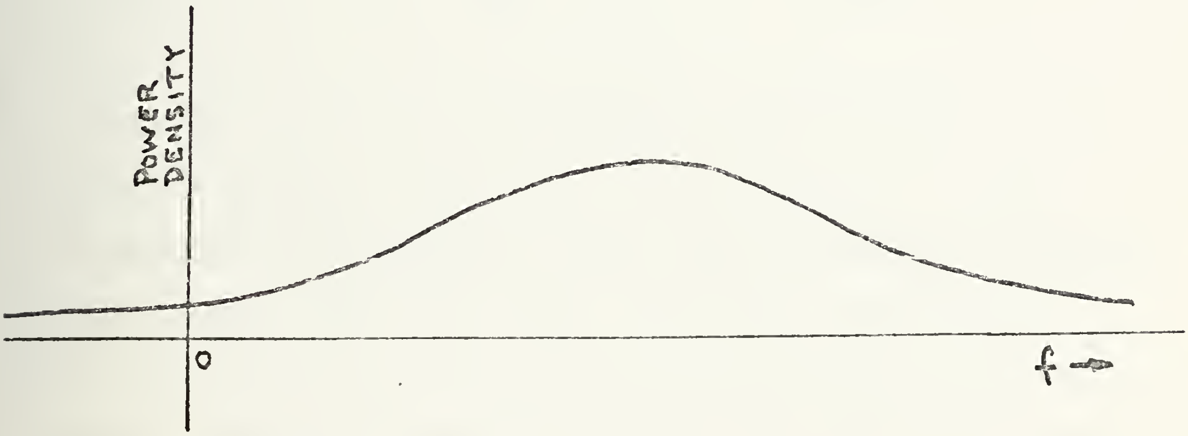
A signal which is passed through a band-pass filter appears as in the next diagram.



As indicated earlier, filters eliminate as much of the noise as possible. How does the filter actually improve detection capability? To answer this question, we examine the way filters help improve detection in a large class of detectors, known as energy detectors. We discuss the way these operate in greater detail a little later, but the general principle of operation

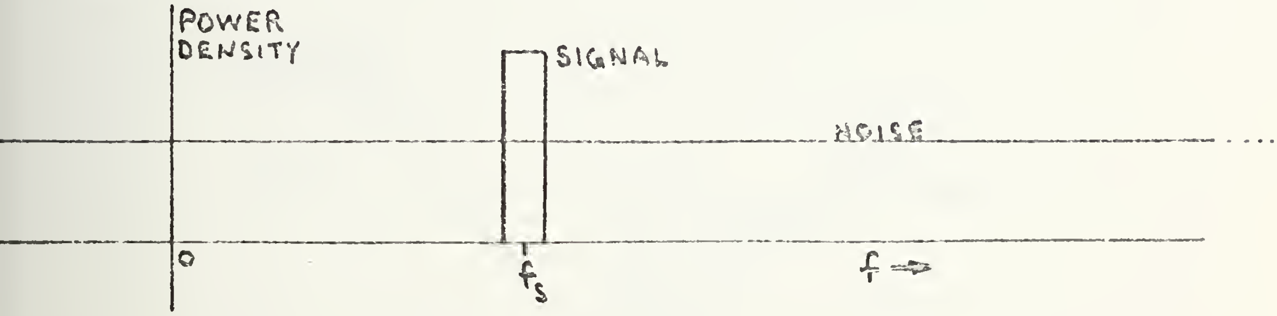
is that the detector senses the total power (or energy) present in the input, and compares this to a "threshold" value. If the input exceeds threshold, the detector indicates the presence of a signal, and if the input does not exceed threshold, it indicates no signal present.

In order to see better how filtering helps to detect signals in noise, consider the "power spectral density" plot of an input. The power spectral density is the power in a signal that is carried by each frequency component.

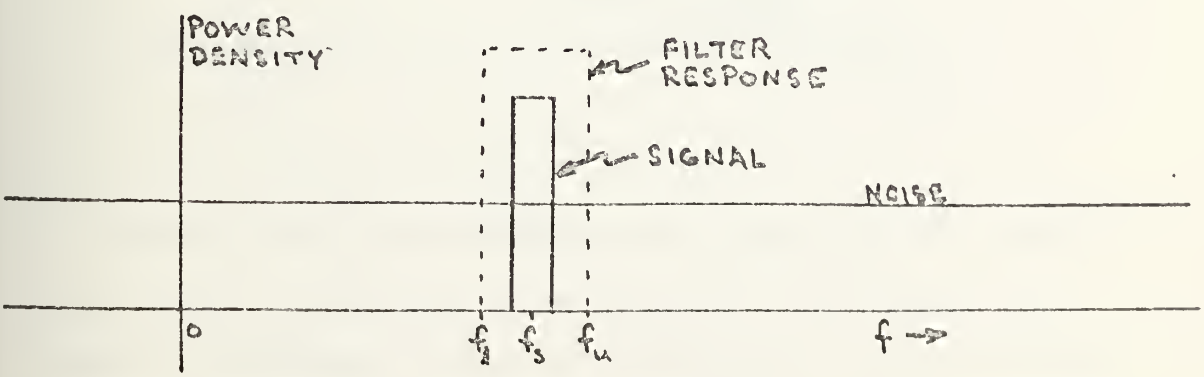


The total power in a signal then corresponds to the integral of the power spectral density over all frequencies. The power spectral density thus gives an indication of which frequencies carry most of the power in a signal, and the relative amount of power at a given frequency compared to that at others. The power in a given frequency band is then proportional to the area under the power spectral density curve between the lower and upper limits of the band. Comparison of the power present

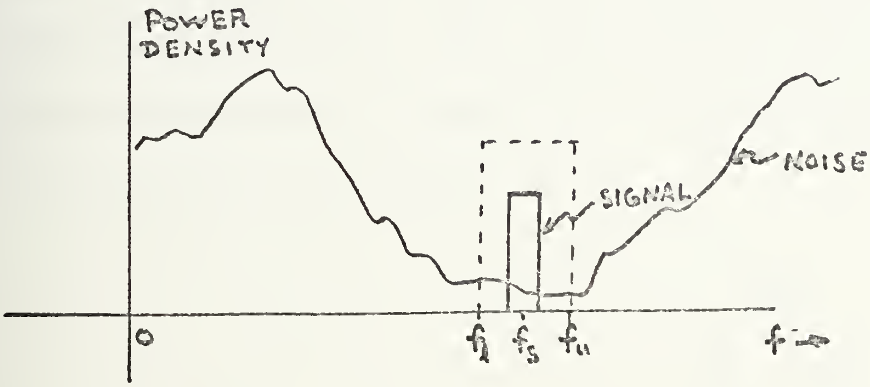
in a signal to the power of the background noise present indicates the detectability of the signal. Consider a signal against a background of "white noise" (white noise is defined as having equal amounts of all spectral components) to see the gains from filtering.



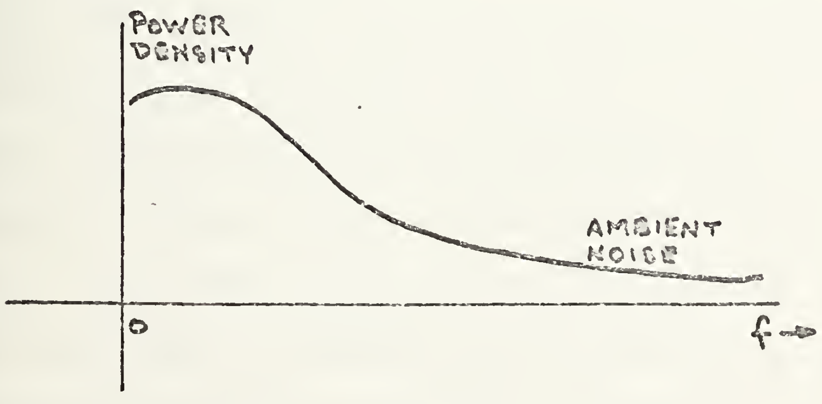
We can see that if the detector examines the entire frequency spectrum and senses the total power present to compare to a threshold value, the presence or absence of the signal does not affect the input significantly. On the other hand, if we filter the input and only allow a narrow band of frequencies about the frequency of interest as the input to the detector, then the presence of a signal will substantially change the amount of power in the input.



This, in effect, raises the signal-to-noise ratio, and thus allows us to detect weaker signals than we could detect before filtering. In the case of non-white noise, the improvement in signal-to-noise (S/N) ratio can be even greater.



Ambient noise in the ocean is an example of non-white noise, and careful filtering will improve detection probability immensely.

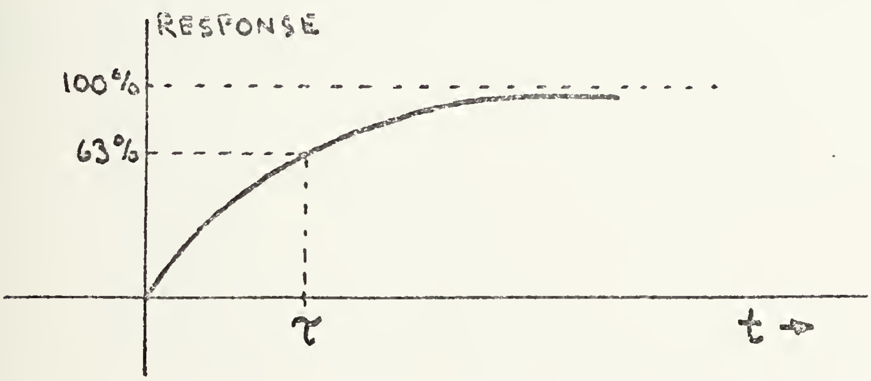


Energy detectors integrate the energy in the input over some time period, some averaging the energy and some not averaging. The value obtained in either case is compared to the threshold, which is set at the value

corresponding to ambient noise when the signal of interest is absent. Therefore, one must determine how long to integrate the input before comparing to threshold. How is this determination accomplished? We saw in the section on Fourier transforms that there was a relationship between the period of a signal and the separation between the frequency components that make up the signal, namely, the separation between components is inversely related to the period of the signal.

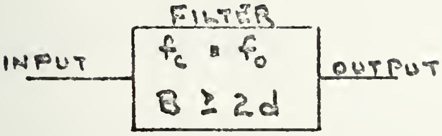
Thus if we assume that the integration time we choose is the period of the input, and that the input exactly repeats itself every period ad infinitum, then the minimum resolvable frequency difference between components of the input is inversely related to the integration time. The longer the input is integrated, the more accurately the frequency components present are determined. If we desire to resolve frequencies 1 Hz apart, we must integrate for at least 1 second, for frequencies 0.1 Hz apart, at least 10 seconds.

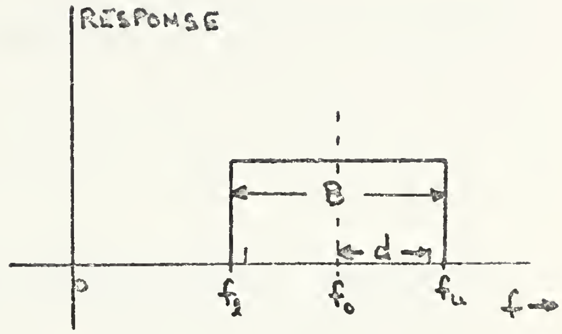
Another consideration in detection system design is the fact that most common filters in military applications today are composed of reactive elements, such as RC/LC/RLC circuits, crystals, or transducer elements, which have finite time constants associated with them. What does this mean? It means that these systems are designed to resonate at a particular frequency or over a band of frequencies, and to block or short out others. These reactive elements require a finite time to "ring up", or establish resonance. Thus, systems do not respond immediately to inputs, and this must be considered in the system design. The response of these circuits has the following appearance.

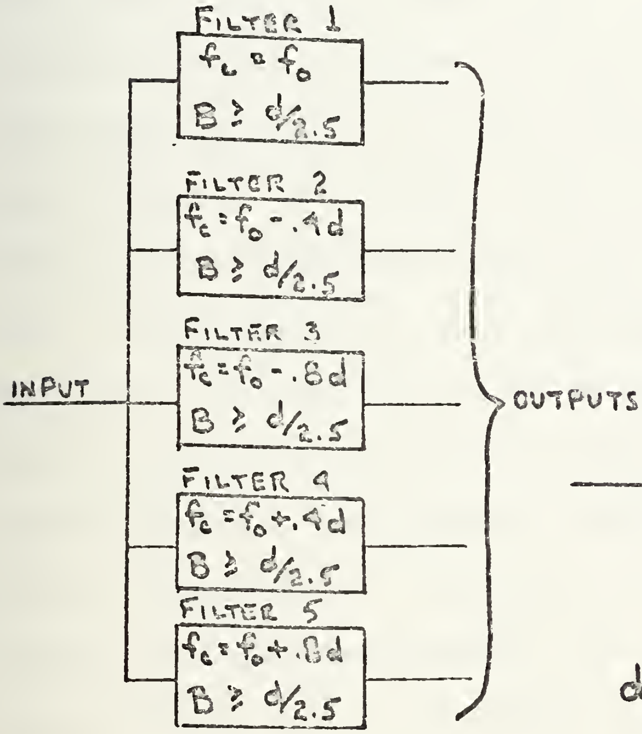


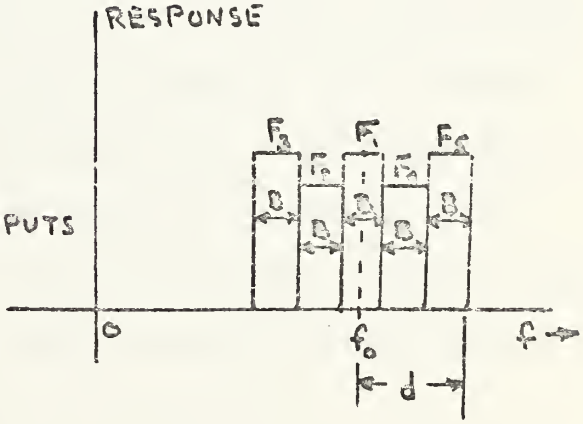
The time required for the response to rise to 63% of the maximum is known as the time constant of the system. The narrower the resonance, that is, the narrower the filter bandpass, the longer the time constant. Thus, if we desire to resolve frequencies to 0.1 Hz, we must allow enough integration time for the filter to respond to each frequency, and this will be longer than the time required to resolve frequencies to 1 Hz.

Another consideration in the design of active ranging detection systems is the requirement to accomodate doppler in the returning echoes. The input filter must have a bandwidth wide enough to pass echoes that are doppler shifted by the maximum amount anticipated during operation of the system, or the echoes will not be detected. If a filter of sufficient bandwidth to ensure this does not give a sufficient S/N ratio improvement, one may alternatively use a bank of narrow band filters centered at various frequencies corresponding to different doppler shifts. This is illustrated in the diagram on the next page.









d = MAXIMUM DOPPLER SHIFT ANTICIPATED

f_0 = TRANSMITTED FREQUENCY

V. RANDOM SIGNALS, POWER SPECTRAL DENSITY, AND NOISE

A. RANDOM SIGNALS

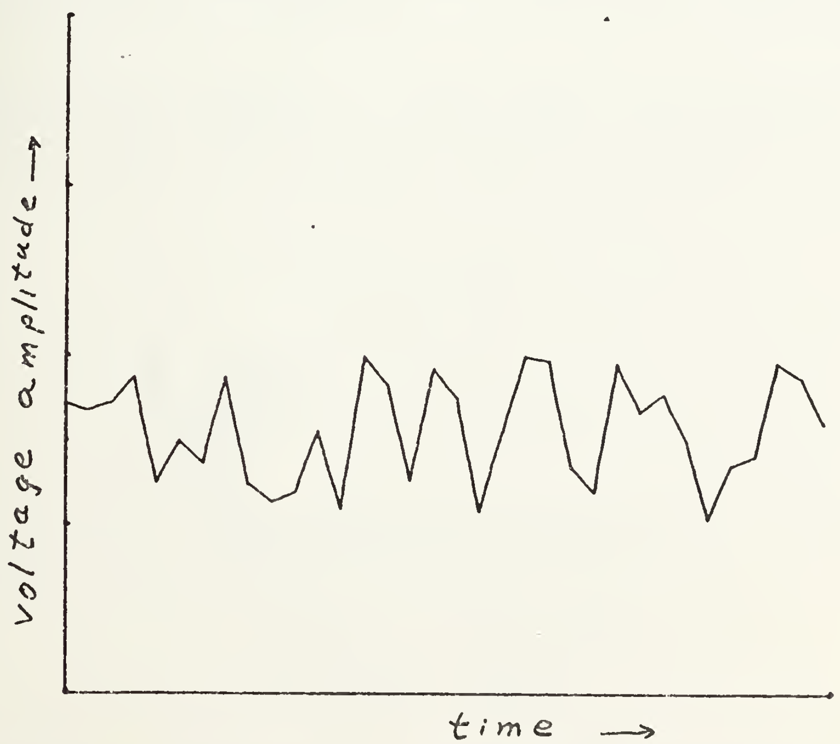
Heretofore the signals discussed have been explicitly described "deterministic signals", such as a square wave, sinusoidal wave, sawtooth, etc. In describing a signal explicitly as sume function of time, it is assumed that its amplitude and phase are known exactly for all time; i.e., past, present, and future. No real world signal meets these criteria, for then the signal would convey no new information, as it could be predicted exactly for any future time. Therefore, consideration must be given to random signals. Random signals of interest to the Antisubmarine Warfare specialist emanate, for example, from various machinery and flow noises. Noise in the ocean due to waves, wind, biological sources, seismic disturbances, etc., is a special case of random signal, and it is always present to some degree whether or not a "target" is present. Processing equipment must be able to distinguish between the random signal of interest, unwanted signals, and noise.

A random signal, unlike a deterministic signal, may <u>only</u> be described by its statistical characteristics. Its past history and its predicted future will be stated in terms of some <u>average value</u> and a given <u>probability</u> that the signal

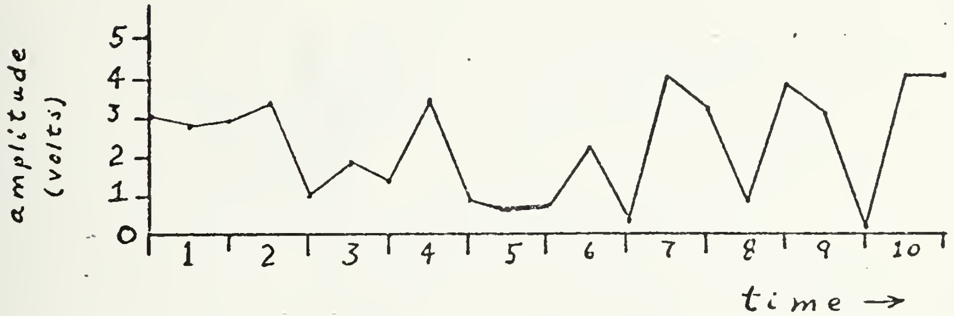
will be within certain limits at a specified time. This "given probability" is represented by a probability density function (pdf).

What is a probability density function? A function is a descriptor, i.e., it usually describes something. Density is a measure of quantity contained within a specified space. Probability refers to the chance, or likelihood, of the occurrence of an event. Therefore, a probability density function describes the likelihood of some quantity being contained within a specified space.

How does the probability density function relate to a random signal? Assume a random signal, x(t), which has some randomly varying voltage amplitude over time.

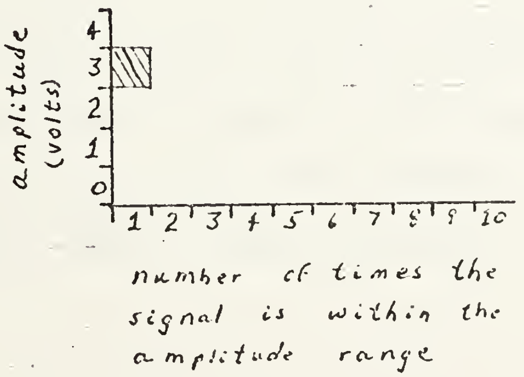


Plot the signal with a vertical scale representing voltage amplitude and the horizontal scale representing time.



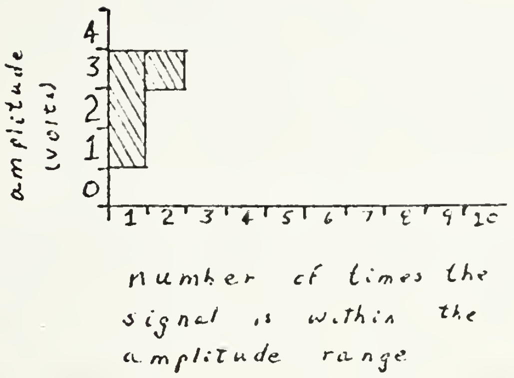
Suppose now that 1-volt represents any value between 0.5 and 1.49; 2-volt represents 1.5 to 2.49; 3-volt represents 2.5 to 3.49, etc. From the random signal, a plot is now made with the vertical scale still representing voltage amplitude but with the horizontal scale representing the number of times the signal is at that amplitude, or within that amplitude range.

During time interval 1, the signal was present only in the 3-volt range and that one block is shaded in on the plot.

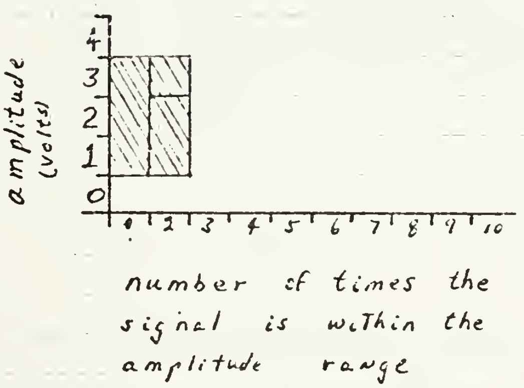


During time interval 2, the signal is present in the 1, 2, and 3-volt ranges. Those additional blocks are shown in

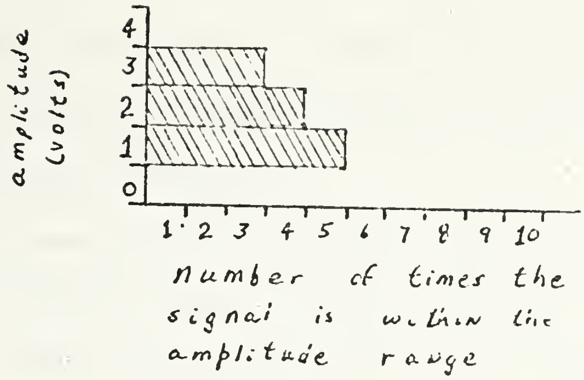
the next diagram.



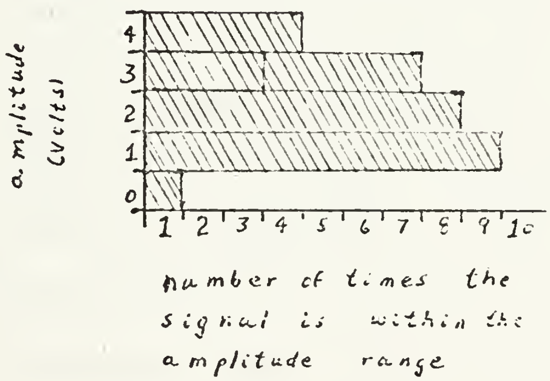
During time interval 3, the signal is present in tne 1 and 2-volt ranges. Those additional blocks are shaded in.



The development should start to become apparent. Look now at time intervals 4,5,and 6. The signal is present in the 3-volt range once, the 2-volt range twice, and in the 1-volt range three times. Shade those in.



To finish the plot, look at time intervals 7,8,9 and 10. The signal appears in the 4-volt range four times; in the 3-volt range three times; in the 2-volt range four times; in the 1-volt range four times; and the 0-volt range once. Shade them in.



The plot above represents the amplitude ranges that the signal was in for the entire time it was observed. The total number of shaded blocks represents the number of times the signal is within the various amplitude zones during the various time intervals for the entire duration of the signal: the 29 blocks account for 100% of signal time. (Count them!) Each shaded block is regarded as a sample value. If the number of sample values is denoted by n, then n = 29 in this example. The percentage of time the signal is in a

particular voltage range can be approximated by dividing the number of signal sample values in that voltage range by the total number of sample values, n.

0-volt range:	1/29 =	3%
1-volt range:	9/29 =	31%
2-volt range:	8/29 =	28%
3-volt range:	7/29 =	24%
4-volt range:	4/29 =	14%
	Total	100%

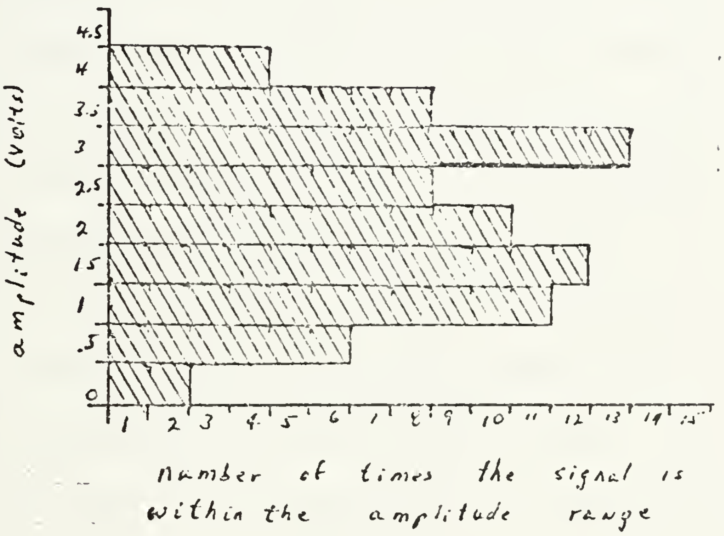
The percentage of time the signal spends at other ranges can also be calculated. Suppose, for example, that in an ASW system there is a detector which detects only signals with amplitudes of 3 volts or more (the detector has a "detection threshold" of 3 volts). To determine the percentage of time the above random signal is detectable, it is necessary to determine the percentage of time it remains in the range of 3 or more volts. The 3-volt range covers from 2.5 to 3.49 volts. Take one-half the time the signal is in the 3-volt range and add it to the entire time the signal is in the 4-volt range to approximate the total time the signal is at or above 3 volts.

$(1/2)(7) + 4 = 7.5$ Divide by n to get percentage.

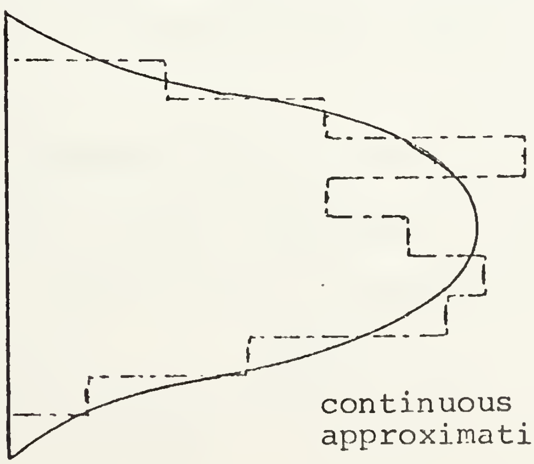
$7.5/29 = 26\%$ Therefore, the signal is detectable 26% of the time.

The accuracy in the determination of the percentage of time the signal is within any specified frequency range can

be increased by making the voltage amplitude ranges smaller and shortening the duration of the time intervals. The next diagram shows sample values for the same signal, but with amplitude ranges of 1/2-volt and the time interval duration halved. It is constructed in the same amnner as the previous plot. A check to verify its accuracy will help you evaluate your understanding of the procedure.



As amplitude range and the time interval length are made smaller and smaller, the plot approaches a continuous curve.



continuous curve approximation

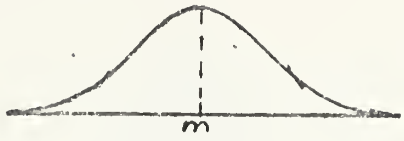
In most practical applications a formula for the continuous curve can be determined, and the corresponding function integrated over the amplitude range of interest to find the percentage of time the signal was within that range.

The probability density function of a signal represents the "probability" that a signal will be within an amplitude range. As the plot constructed above represents 100% of the time of the signal, so too does the probability density function represent the total probability of the signal. It is scaled from 0 to 1, i.e., the area under the probability density function curve is equal to 1. If the detector in the example just mentioned detects, with certainty, any voltage amplitude of 3 volts or more, and the signal spends 26% of the time at 3 volts or more, then the probability that the signal will be detected in the 3-volt or more range is 0.26. The probability density is expressed by a function which may be integrated to find the probability that the signal is in any prescribed amplitude range. For probability density functions used in many applied problems, the integration process can be avoided, sincd the results are readily available in standard tables.

The probability density function (pdf) most commonly used for electrical signals is the "gaussian" or "normal" pdf. It is the familiar bell-shaped curve described by the equation

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-m)^2}{2\sigma^2}} .$$

In the equation for p(x),

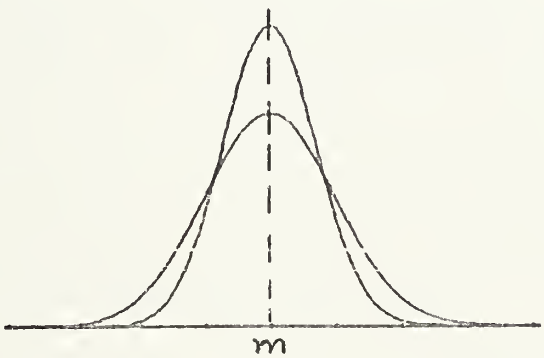


m = mean, or average, value,

x = value of the signal, x(t), at time t,

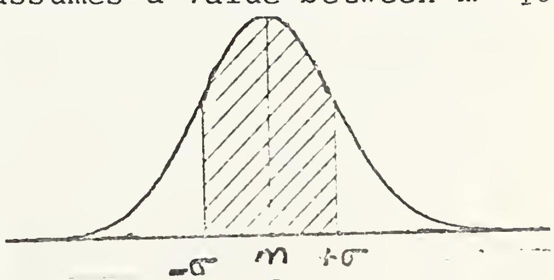
σ = standard deviation (σ is the Greek lower case sigma).

The standard deviation, σ, is a measure of the spread of the signal abour its average value, m. In the next figure we we show two gaussian curves with the same mean, but different values of σ. The area under each curve is equal to 1.0.

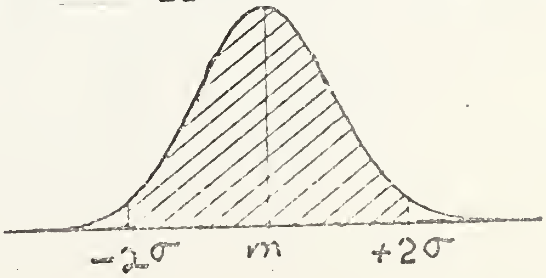


For the gaussian pdf, about 68% of the area under the curve lies between m - σ and m + σ, which means that the probability that the random signal, x(t), assumes a value between m -1σ and m + 1σ is 0.68.

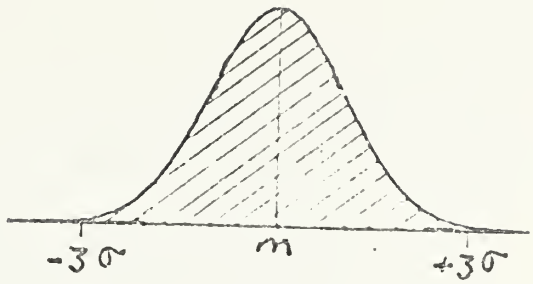
m ± 1σ, P(x) = 0.68
(Shaded area = 68%
of total area)



m ± 2σ, P(x) = 0.95
(Shaded area = 95%
of total area)

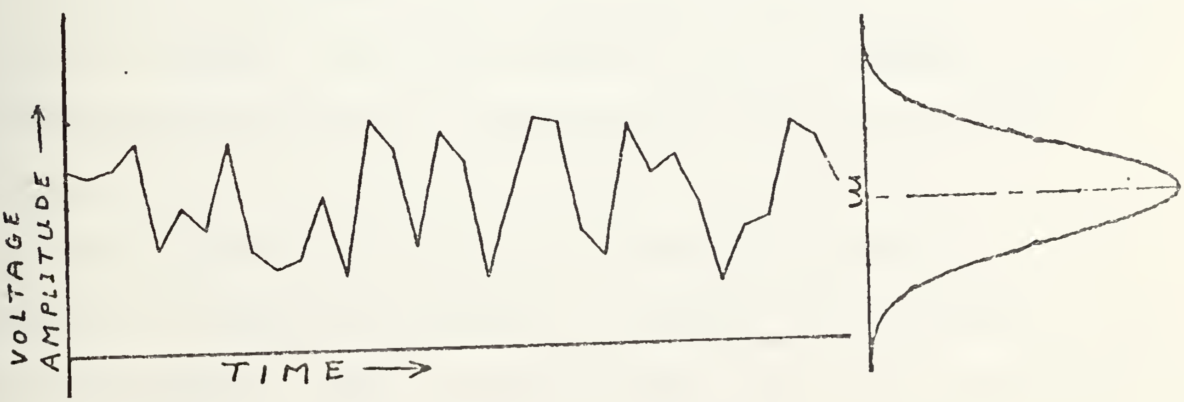


$m \pm 3\sigma$, $P(x) = 0.997$
(Shaded area = 99.7% of total area)



In the illustrations on the previous page, only 0.997 of the total probability has been accounted for. This is because there are no limits on a gaussian pdf; the range is unbounded. A signal may conceivably exist at any finite amplitude above or below the average value. The probability that the signal will occur outside the $m \pm 3\sigma$ range is only 0.003 (1.0 - 0.997 = 0.003). Limiting circuits are normally used early in the processing stage to prevent the rare occurrence of a large amplitude signal from damaging equipment. The important part of the signal, 0.997 of it, is still retained.

The gaussian pdf is centered around an average value. For an electrical signal this is the average value of the amplitude of the signal. Look at the signal and its pdf side by side with the pdf oriented to the signal amplitude.



The value chosen for m is the statistical average of the 29 sample values above. Each of the sample values was 0, 1, 2, 3, or 4. The average, m, is equal to the sum of the individual sample values divided by the sample size, n.

$$m = \frac{0+1+1+1+1+1+1+1+1+1+2+2+2+2+2+2+2+2+3+\ldots+4\ldots}{29}$$

or, since many values are the same,

$$m = \frac{1(0) + 9(1) + 8(2) + 7(3) + 4(4)}{29} = 2.14$$

Algebraically expressed, this is $m = \frac{1}{n} \sum_{j=1}^{n} x_j$ where the $\sum_{j=1}^{n}$ means the sum of all the samples, x_j, from j = 1 to j = n, or $x_1 + x_2 + x_3 + \ldots + x_n$. If the number of sample values becomes very large, the sum of the products of the individual sample values and their probability of occurrence becomes a better choice for m. Increasing the number of sample values gives a more accurate description of the signal. If m is calculated for the plot with the amplitude range and time duration halved, n = 74 and m = 2.095. The algebraic expression, if n is very large, is approximately

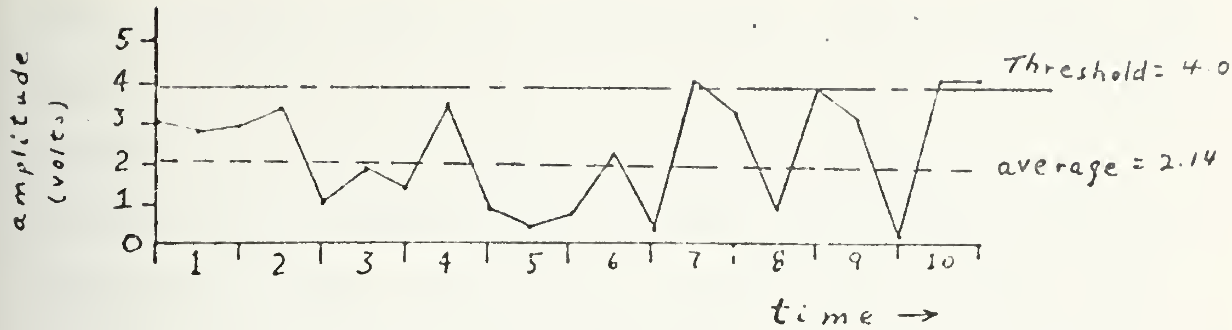
$$m = \sum_{j=1}^{\infty} (x_j) \cdot P(x_j).$$

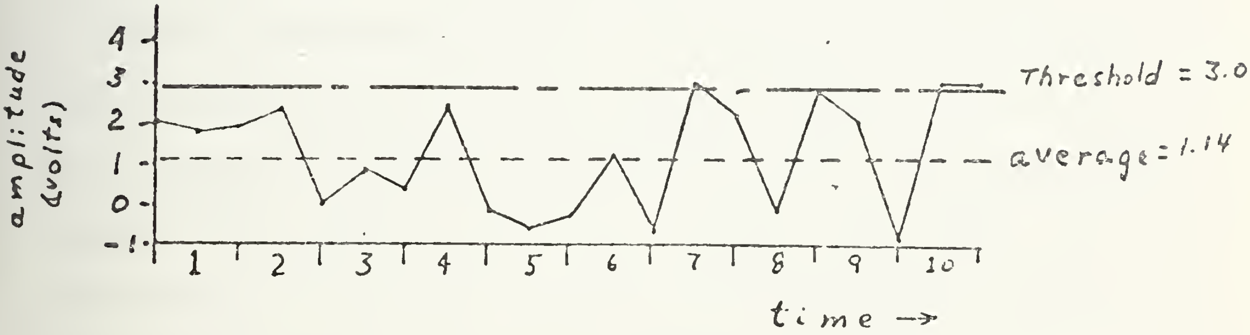
It has been shown how statistical characteristics may describe a signal. The next problem is how to <u>obtain</u> signal characteristics that are <u>statistically valid</u> to identify a signal.

The validity of statistical methods in determining the outcome of a process depends on repition of the process over and over again. With a random signal, x(t), this repetition may be accomplished by using an "ensemble" of random signal

generators with each signal generated having the same broad statistical characteristics. If the individual outputs from this "ensemble" are measured <u>simultaneously</u> at some time, t, and averaged, the result would be an "ensemble average", indicated by $\overline{x(t)}$. This "ensemble average" is identical to, and has all the properties of, a statistical average. However, most signals of interest in an ASW environment originate from a single source. Therefore, only the time average of the signal, $\langle x(t) \rangle$, may be obtained.

The time average, $\langle x(t) \rangle$, and the ensemble average, $\overline{x(t)}$, are not always the same. For example, suppose the statistical characteristics of the signal from the random signal generators in the ensemble vary with time. Such a variation would not be reflected in measurements made at a fixed time, t_1. The ensemble average at time t_1 would be different from the ensemble average at time t_2. A random signal with these characteristics is called "non-stationary". If a detector is used to detect the random signal in our example above, what is the effect on the detector if the random signal is "non-stationary"? The average, m, was calculated as 2.14. Suppose that m varies with time, and that over another time interval m is calculated to be 1.14. If the signal over the second time has the same characteristics as over the first, the effect on detection would be the same as raising the threshold to 4 volts over the first interval, as indicated in the next diagram.



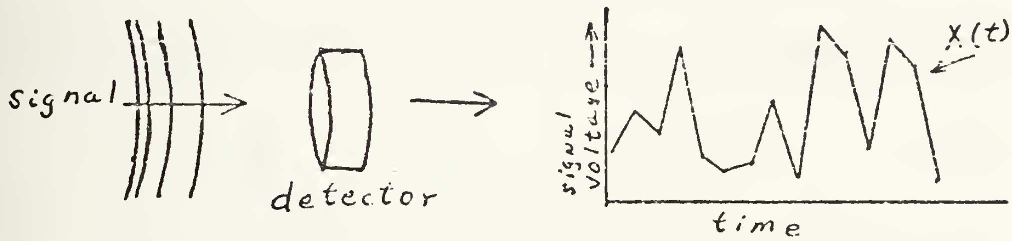


The probability that x = 3 volts or more would become 2/29 = 0.069 vice 0.26 for m = 2.14! If the average is unchanged and the standard deviation increased by a factor of three, the detector would be observing only 0.68 of the signal as opposed to 0.997. Therefore, if the random signal is not "stationary" (i.e., its statistical characteristics do not vary with time) or at least "stationary" for the processing time of the detector, the detector will be very inefficient. Even though the signal may be stationary, the time and ensemble averages may still be unequal.

Fortunately, many random signals that mjst be processed in ASW have time and ensemble averages which are identical, perhaps not for all time, but at least for the processing time required. Signals with their time and ensemble averages

equal, $\langle x(t) \rangle = \overline{x(t)}$, are said to come from an "ergodic process",or to have the property of "ergodicity". An "ergodic process" is also stationary. Under the assumption that a signal from an ergodic process means the time and ensemble averages are equal, the signal is stationary and the time average will have all the properties of a statistical average.

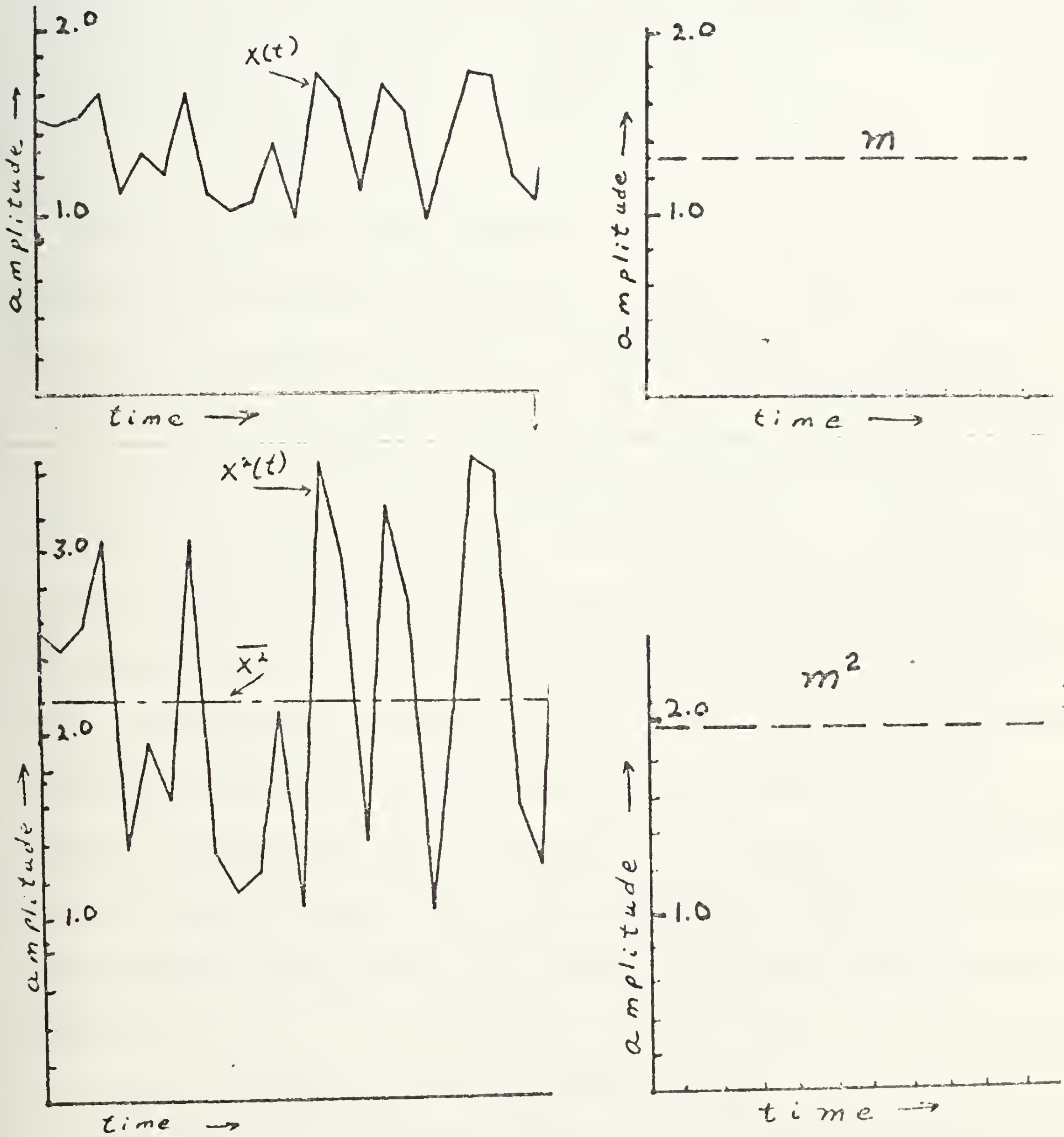
Having extablished that a time average will be statistically valid if it is from an ergodic process, how do we obtain the other statistical characteristics which describe a signal? All acoustic detectors convert the signal (and, unfortunately, the noise, which will be addressed later) into some time-varying voltage or current, x(t).



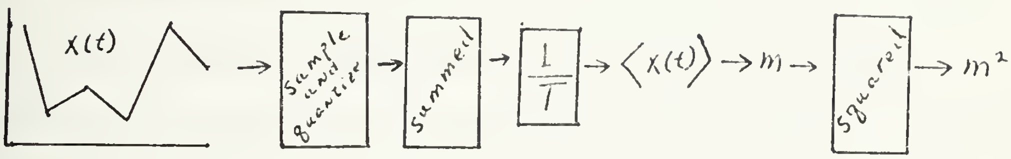
The time average of the signal, $\langle x(t) \rangle$, assuming ergodicity, is equal to the average value, m. This average value is both the value about which the signal's gaussian pdf is centered and, as the time average, represents the "dc" component of the signal (direct current, also called the steady-state or the non-time-varying component). If m = 0, there is no "dc" component. If the signal, x(t), is impressed across a one-ohm resistor, the instantaneous power dissipated is $x^2(t)$. The time average, $\langle x^2(t) \rangle$, becomes the "second moment", which represents the total average power, $\overline{x^2}$, of the

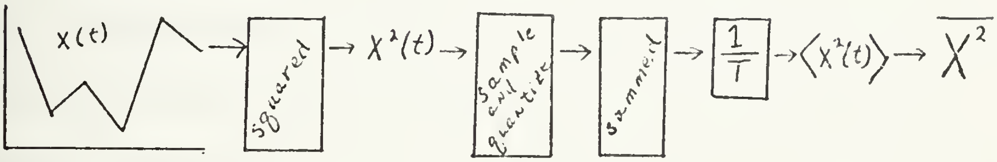
electrical signal. The "second moment" differs from the "average value squared", m^2, which is the dc power, or the power in the non-time-varying component: m^2 is obtained by squaring after averaging, whereas $\overline{x^2}$ is obtained by squaring before averaging.

As an illustraion of these values, consider a random signal, x(t).



The following drawings illustrate computation of m^2 and $\overline{x^2}$.





There sre two other measures which identify a specific gaussian pdf besides the average value, m, about which it is centered. A random signal is composed of a dc component and an "ac" component (alternating current, or the time varying component of the signal). The total power, $\overline{x^2}$, must be the sum of the dc power, m^2, and the ac power, σ^2. Though representing the ac power of the signal, σ^2 is the "variance" of the gaussian pdf. The variance is equal to the second moment minus the average squared, $\sigma^2 = \overline{x^2} - m^2$. For an electrical signal the variance is the total average power minus the dc power. As the standard deviation, σ, represents the square root of the ac power, it is more commonly known as the root-mean square (rms) value. We summarize these definitions briefly.

Given an ergodic random signal, x(t):

1. the average (mean) value, m, is its dc component;

2. the average-squared, m^2, is its dc power;

3. the second moment, $\overline{x^2}$, is its total average power;

4. the variance, $\sigma^2 = \overline{x^2} - m^2$, is its ac power;

5. the standard deviation, σ, is its rms value.

In "Fourier Transform Properties", the concept of correlation of a signal was discussed. It is considered again here because of its unique properties, and because of its relationship to the "power spectral density" of a random signal. Those who feel they have not grasped the concept of correlation should review that section before proceeding.

The autocorrelation function, $R(\tau)$, was defined previously as the time average, $R(\tau) = \langle x(t) \cdot x(t + \tau) \rangle$. For an ergodic random signal the autocorrelation function becomes:

$$R(\tau) = \langle x(t) \cdot x(t + \tau) \rangle = \overline{x(t) \cdot x(t + \tau)}$$

The value of $R(\tau)$ is a function of the time delay, τ. At $\tau=0$, $R(\tau)$ is a manimum:

$$R(0) = \overline{x(t) \cdot x(t+0)} = \overline{x^2} = \text{total average power.}$$

As increases, $R(\tau)$ decreases, until at $\tau=$ infinity, $R(\tau)$ is a minimum:

$$R(\infty) = m^2 = \text{dc power;}$$

$$R(0) - R(\infty) = \sigma^2 = \overline{x^2} - m^2 = \text{ac power.}$$

The autocorrelation function is also a measure of the "time coherence" of the random signal. If τ is small, $x(t)$ and $x(t+\tau)$ will be close together in time. Therefore, the presence of one will have some effect on the presence of the other, a condition known as "statistical dependence". If the

presence of $x(t)$ has no effect on $x(t+\tau)$, such as happens if τ is large and $x(t)$ is random and non-periodic, then $x(t)$ and $x(t+\tau)$ are said to be "statistically independent". If for a signal with a gaussian pdf the absolute value of the autocorrelation function minus the dc power equals zero, $|R(\tau)| - m^2 = 0$, then $x(t)$ and $x(t+\tau)$ are statistically independent and "uncorrelated". Statistical independence simplifies the calculation of the autocorrelation function. Whether or not two signals are "correlated" becomes important in the calculation of their average powers.

Assume that a signal has been formed by the addition of two separate signals: $z(t) = x(t) + y(t)$. The correlation function of $z(t)$ will be of the form

$(x+y)(x+y) = x^2 + xy + yx + y^2$, and is equal to:

$$R_z(\tau) = \langle x(t)\cdot x(t+\tau)\rangle + \langle y(t)\cdot y(t+\tau)\rangle + \langle x(t)\cdot y(t+\tau)\rangle + \langle y(t)\cdot x(t+\tau)\rangle$$

$$R_z(\tau) = R_{xx}(\tau) + R_{yy}(\tau) + R_{xy}(\tau) + R_{yx}(\tau)$$

where the last two terms are the "cross correlation terms" of $x(t)$ with $y(t)$ and $y(t)$ with $x(t)$. As noted previously, the above correlation functions represent the average power in the signals, and can be written as:

$$P_z = P_{xx} + P_{yy} + P_{xy} + P_{yx} .$$

If the two signals, $x(t)$ and $y(t)$, are correlated over the time delay, τ, then the total signal power, S, is the sum of the signal power in signal $x(t)$, S_{xx}, the signal power in signal $y(t)$, S_{yy}, <u>and the cross correlation</u> of their combined signal powers, S_{xy} and S_{yx}:

$$S_z = S_{xx} + S_{yy} + S_{xy} + S_{yx} .$$

If the signal is the sum of mutually uncorrelated signals, as in the case of noise uncorrelated over the time delay, τ, then the total noise power, N, is the sum of only the noise power N_{xx} and N_{yy}. The cross correlations are equal to zero:

$$N_z = N_{xx} + N_{yy}; \quad (N_{xy} = 0; \quad N_{yx} = 0).$$

For a signal which is the sum of mutually uncorrelated signals, the correlation function is the sum of only the autocorrelations (the cross correlations equal zero), and the average power is the sum of only the average powers of the individual signals (the cross correlated powers equal zero).

B. POWER SPECTRAL DENSITY

Thus far only the time domain aspect of random signals has been addressed. The frequency domain is represented by the power spectral density, which was introduced in "Filters and Bandwidth Requirements". The device used to transfer between the time and frequency domains is the Fourier Transform. The Weiner-Kinchine Theorem states that the power spectral density, G(f), and the autocorrelation function, $R(\tau)$, are Fourier Transforms of each other:

$$G(f) = \mathcal{F}[R(\tau)] = \int_{-\infty}^{+\infty} x(t)\ x(t+\tau) e^{-j\omega\tau} d\tau .$$

And, from the Duality Theorem,

$$\text{if} \quad R(\tau) \leftrightarrow G(f), \quad \text{then} \quad G(\tau) \leftrightarrow R(f).$$

As can be seen above, the random signal may be described in either the time domain, $R(\tau)$, or the frequency domain, G(f),

with free interchange between the two.

Since the power spectral density is the Fourier Transform of the autocorrelation function, the Fourier Transform Theorems may be applied to the power spectral density. The systems designer uses the properties expressed by these theorems to design an efficient detector/signal processing system. The ASW specialist who has knowledge of these properties can better understand how his system is designed to operate and therefore, utilize it more effectively. The duality theorem was already utilized. The other major theorems applicable to power spectral density will be explained along with examples of their usage.

1. <u>Frequency Translation Theorem</u>

If a signal, $x(t)$, is bandlimited in $W < f_c$, and a new signal, $y(t)$, is formed by multiplying $x(t)$ by $\cos(\omega_c t+\phi)$, $y(t) = x(t)\cos(\omega_c t+\phi)$, then the correlation function of $y(t)$ is equal to one-half the correlation function of $x(t)$ $\cos\omega_c\tau$, that is $R_y(\tau) = (1/2)R_x(\tau)\cos\omega_c\tau$. In addition, the power spectral density, $G(f)$, is

$$G_y(f) = (1/4)G_x(f-f_c) + (1/4)G_x(f+f_c) .$$

Multiplication in the time domain by $\cos\omega_c t$ becomes translation in the frequency domain, as the power spectrum is shifted up and down in frequency by $\pm f_c$. The condition that the bandwidth, W, be less than f_c is necessary to ensure that $G_x(f-f_c)$ and $G_x(f+f_c)$ do not overlap, otherwise aliasing, as described in "Filters and Bandwidth Requirements", would result. Notice

also that in multiplication by $\cos(\omega_c t+\phi)$, the phase factor, ϕ, is lost in translation to the power spectral density. Therefore, the original random waveform may not be reconstructed from knowledge of the power spectral density. However, by returning to the time domain via the inverse Fourier Transform, the autocorrelation function, $R(\tau)$, with its significant statistical properties, may be found.

Frequency translation is very useful in signal processing. Suppose that due to space, weight, or power limitations it is impractical to locate the detector and the processor together. The low frequency (0 to 1000 Hz) sound may be detected, translated to radio frequency (RF) (HF = 2 - 30 MHz, VHF = 100 - 200 MHz, UHF = 200 - 400 MHz) and broadcast to the processor. In the same manner, one processor can handle many detectors if each detector's signal is translated to a separate frequency band within the RF range (Detector 1, frequency f_1 to f_2; Detector 2, frequency f_3 to f_4; etc., all within the UHF band). The problem could be one of processing signals of many different frequencies. Rather than having many different processors, each processing only one signal, it may be better to have one processor operating at a fixed frequency. All of the signals could be translated to the processor frequency for processing. If the processor is very elaborate and/or very expensive, this method becomes desirable.

2. Integration Theorem

If $y(t) = \int_{-\infty}^{t} x(t')dt'$, then $G_y(f) = \frac{1}{(2\pi f)^2} G_x(f)$.

The multiplication factor, $\frac{1}{(2\pi f)^2}$, indicates that the <u>low</u> frequency components of the power spectral density will be enhanced by integration. The ocean acts as an "integrating filter", in that it allows the low frequencies to propagate, while rapidly attenuating the higher frequencies. This is one reason why, in an attempt to get long detection ranges, active sonars and passive processing systems have increasingly exploited the lower frequencies.

3. <u>Differentiation Theorem</u>

This theorem is offered without an example since it follows so naturally the integration theorem.

If $Y(t) = d[x(t)]/dt$, then $G_y(f) = (2\pi f)^2 G_x(f)$.

The multiplication factor, $(2\pi f)^2$, indicates that the <u>high</u> frequency components of the power spectral density will be enhanced by differentiation.

C. NOISE

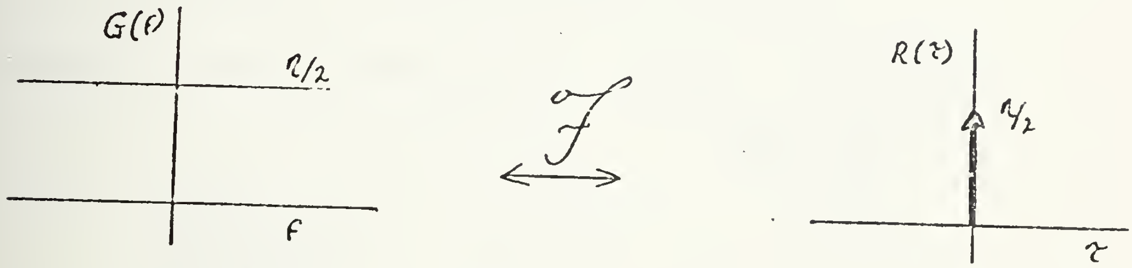
At the beginning of this section, noise was mentioned as a "special case of random signal which is always present to some degree whether or not a 'target' is present". In communications, noise is often defined as any electrical interference, or <u>unwanted signal</u>. Signal then, in addition to the characteristics previously used to describe it, has the quality of being wanted, sought after, emanating from a source of interest, and containing intelligence. In ASW, signal is always associated with "target". Noise, on the other hand,

is anything which interferes with, or tends to mask, the signal. Often "one man's signal is another man's noise" as any patrol aviation Tactical Coordinator whose buoy field has just been penetrated by a "pinging" Destroyer will verify. An acoustic detector converts both signal and noise into a time-varying electrical voltage or current. An examination of some of the characteristice of noise will help in understanding how systems may be designed to reduce it.

Another way to define noise is simply as that quantity observed in the absence of signal. Noise may come from a variety of manmade or naturally occurring sources. Systems are designed to eliminate as much noise as possible, but some noise will inevitably remain. The thermal motion of electrons in the conducting media of the components of a system, for instance, is one unavoidable cause of electron noise. Thermal noise is interesting in that its power spectrum is constant over a wide range of frequencies. It is designated "white noise" by analogy to white light, as all frequency components are present in equal amplitudes. The amplitude of thermal noise has been proven to have a "gaussian" pdf. It is therefore referred to as "gaussian white noise" ("gaussian" amplitude distribution; "white" frequency distribution). If "gaussian white noise" is played over a loudspeaker, it sounds dull and monotonous, somewhat like a waterfall. The subtleties of its random variations are hidden from the human observer.

The power spectrum of gaussian white noise, with zero

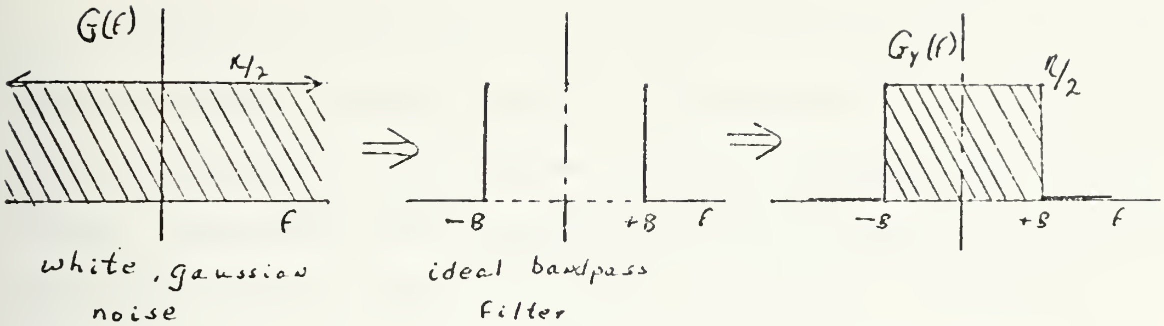
mean, is $G(f) = \frac{\eta}{2}$, where η is power density in watts per Hertz. By Fourier transformation, the autocorrelation function is $R(\tau) = \int_{-\infty}^{\infty} \frac{\eta}{2} e^{j\omega\tau} df = (\frac{\eta}{2})\delta(\tau)$, where δ represents the "delta function" introduced previously.



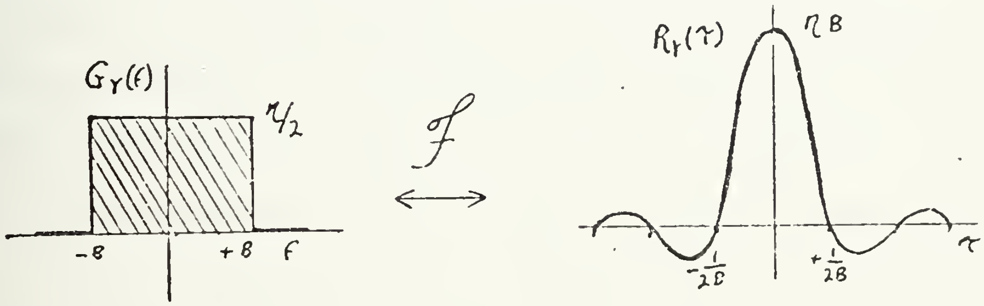
It is apparent from the $R(\tau)$ plot that for any time delay, τ, other than zero, the autocorrelation function is zero; therefore, any two different samples of a gaussian white noise signal are uncorrelated and statistically independent.

The signals of interest in ASW are frequency limited, i.e., there is some finite frequency band within which the signal exists. Filtering in order to improve detection of that signal was discussed earlier. If gaussian white noise is filtered, its frequency components are naturally those of the filter bandwidth, but the amplitude distribution remasins gaussian. The output power spectrum of white gaussian noise, filtered by an ideal filter, is a rectangular function,

$G_y(f) = (\frac{\eta}{2}) \Pi(\frac{f}{2B})$, where Π indicates a rectangular function and B represents the filter bandpass. The autocorrelation function is the inverse Fourier Transform of the power spectral density. As indicated in "Fourier Transform Properties", the Fourier Transform of a rectangular



pulse is a "SINC" function, $R_y(\tau) = \eta B \text{SINC} 2B\tau$



The above figure shows that filtering has produced the following results.

1. The power spectrum, though no longer white, is constant over the finite frequency range of the filter.

2. The output power is finite, $\overline{y^2} = \frac{\eta}{2}(2B) = \eta B$. It varies linearly with bandwidth, B.

3. The output signal is correlated over time intervals of about $\frac{1}{2B}$.

How is this knowledge used in signal processing?

Item (1). The noise in which a signal of interest must be detected is often not gaussian white noise. However, through judicious choice of a bandwidth, its power spectral density may be constant over that range. Otherwise, a pre-whitening filter may be used to make the noise power spectral density constant over the bandwidth. Therefore, after filtering it may be treated as gaussian white noise.

Item (2). The noise power after filtering is $\overline{y^2} = \eta B$. It is a direct, linear function of bandwidth, B. By narrowing the bandwidth, the amount of noise power present is reduced. Therefore, the signal power is enhanced in relation to the noise power. This is crucial in a power, or energy detector.

Item (3). The bandwidth of the filter determines the minimum time delay, τ, in a correlation detector. It must be greater than $\frac{1}{2B}$, ($\tau > \frac{1}{2B}$), or the noise, as well as the signal, will be correlated. If the noise is uncorrelated, which it will be for $\tau > \frac{1}{2B}$, only its average power will add in a correlation detector, thus decreasing in relation to the signal power.

The bandwidth of the filter also determines the finite integration time to be used in determining the time average. The time average is indicated by $\langle\rangle$ (brackets) and is defined as:

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\ x(t+\tau)\ dt.$$

Replacing T by $\frac{1}{B}$ and by $\frac{1}{2B}$, we obtain

$$B \int_0^{\frac{1}{2B}} x(t)\ x(t+\tfrac{1}{2B})\,dt.$$

A review of the development of bandwidth versus integration time in the "Filters and Bandwidth Requirements" section may be helpful in the understanding of these relationships.

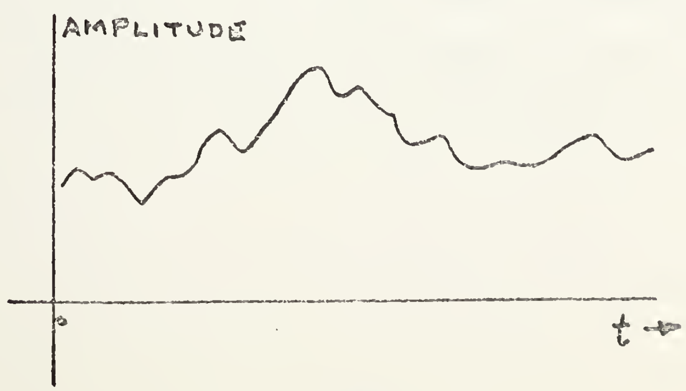
This section has been an attempt to describe the concepts of random signals, power spectral density, and noise. The intent has been to provide a basis for understanding their

individual characteristics. A grasp of these characteristics is necessary to understand the methods of signal processing. The most common methods in current use, DELTIC, energy detectors, correlation detectors and beam formers, will be presented in following sections.

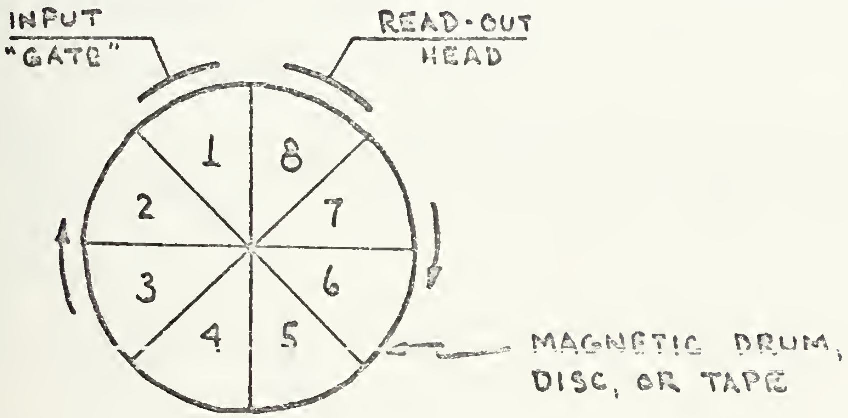
VI. THE DELTIC

In the section on filters and bandwidth, the relationship between integration time and frequency resolution was discussed. It was shown that one must integrate for longer periods of time as the frequency resolution requirements become stricter. In many cases, however, the integration time required to give the necessary resolution is much too long to be of tactical use. In order to circumvent this, several processing schemes have been developed which in effect give more resolution for a given integration time. One of the more common in use with military processors is known as the DELTIC (DELay TIme Compressor). Let us investigate the manner in which this processor accomplishes this feat.

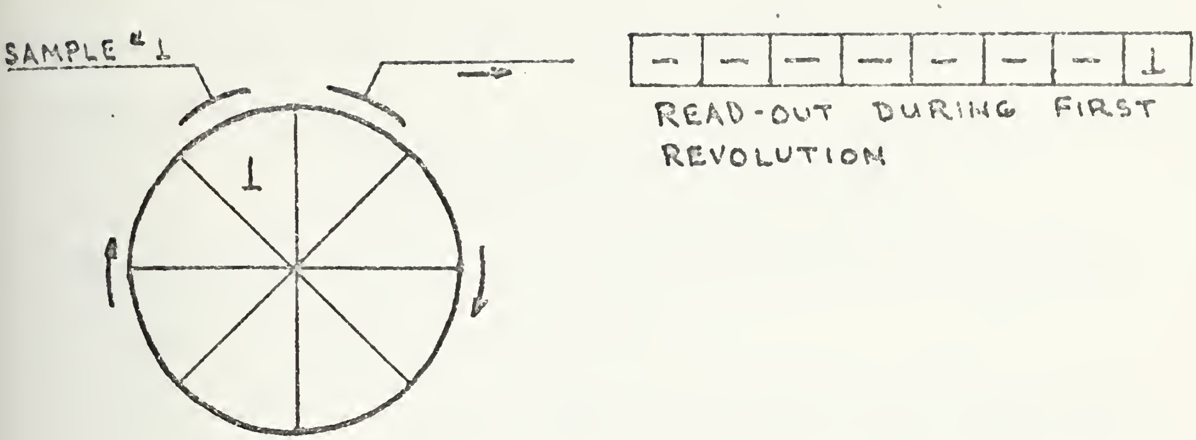
It has been shown that processing the signal as it is received by the detector will not give any greater resolution. The DELTIC takes the signal as received, and records it on a magnetic tape, disc, or drum. It does this in a special sequence, however. To see this, let us take a simple signal and observe how it is processed.



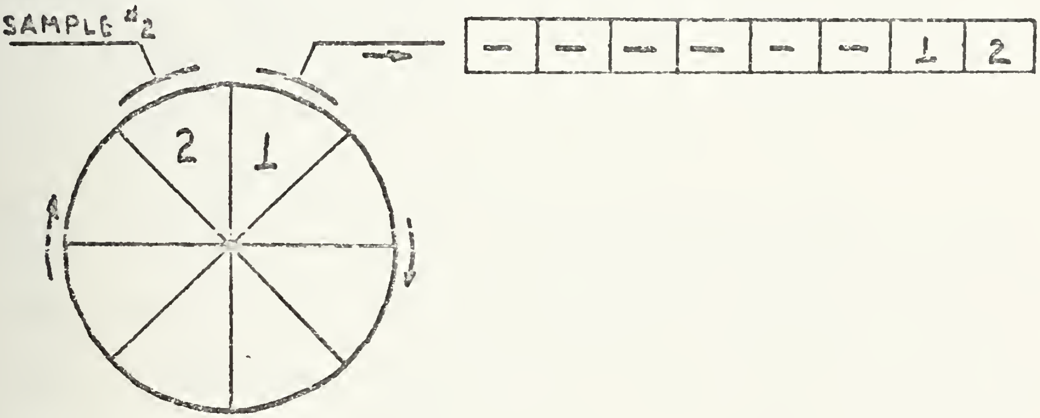
For simplicity in showing the operation, an 8-sample point "time window" will be used. In actual practice, 1000 or more sample points are commonly used. If the time between sample points is τ, the time window observed will be $8\tau = T$. Therefore if the DELTIC is not used, the best resolution we can get is $1/8\tau$, and it would require 8τ seconds to get this resolution. The physical layout of a DELTIC is shown in the next diagram.



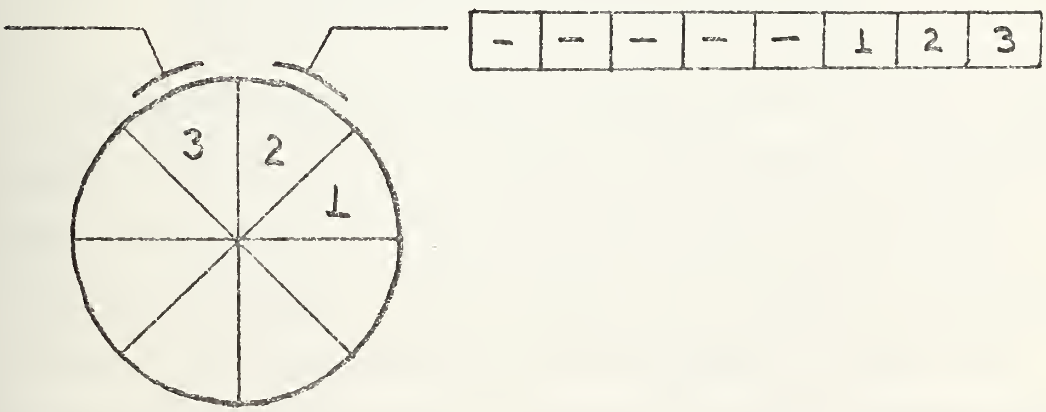
The drum is rotated at a rate such that it makes one revolution plus one sample space during the time period τ. In this case, with 8 sample points it makes 1.125 revolutions per τ seconds. The read-out head continuously reads out the sample values as the drum rotates under it. The input gate injects a new sample point once each 1.125 revolutions in such a way that the sample point which has been on the drum the longest time is replaced. Starting with a blank drum, let us examine the process as it occurs.



When the drum has made 1.125 revolutions (in   sec.) the input gate inserts sample point #2.



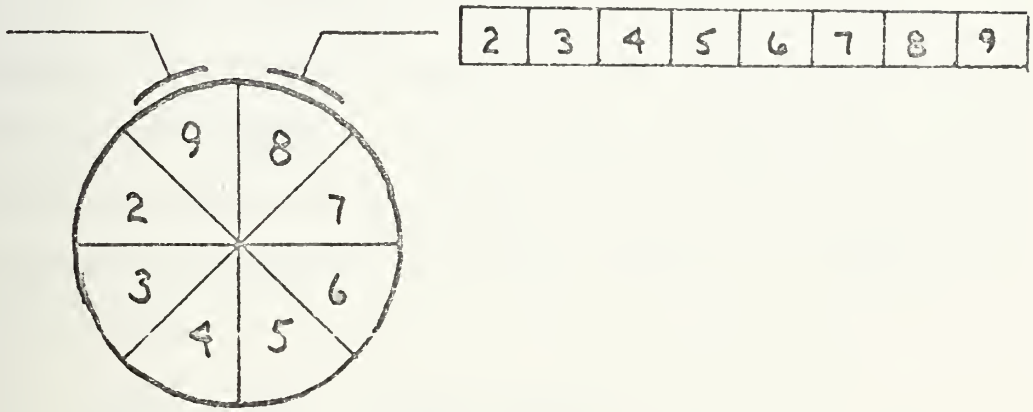
After 1.125 more revolutions, the 3rd sample point is inserted.



The remaining spots are filled similarly until all 8 are filled. The output during one revolution of the drum then is:

1	2	3	4	5	6	7	8

When the next sample point is inserted after 1.125 revolutions, it displaces the first.



Once the drum is filled (which takes 8τ sec.), a new value replaces the oldest one every τ seconds. If we look at the output from the read-out head, we see that there are 9 values read out each τ seconds (8 values/revolution, 1.125 revolutions/τ seconds).

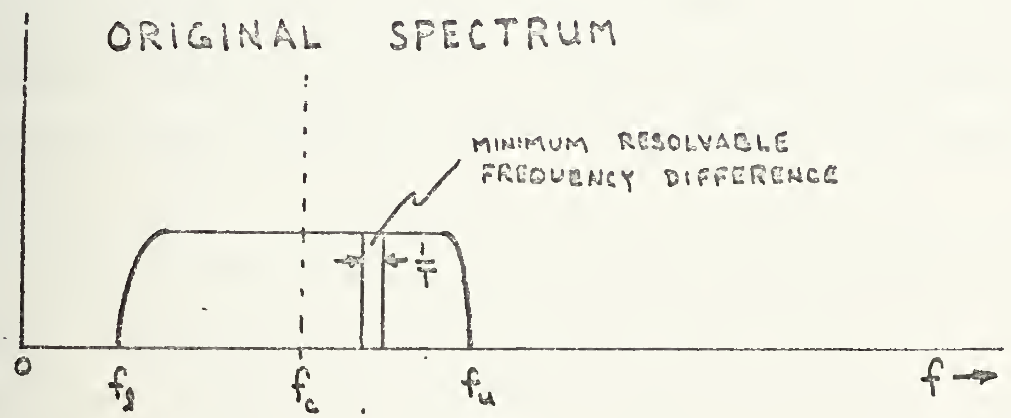
1	2	3	4	5	6	7	8	1

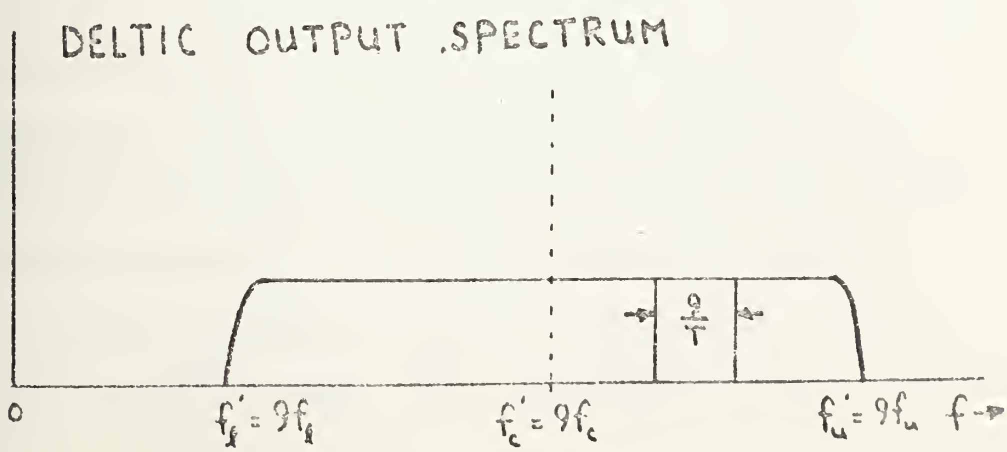
τ sec.

The output of the DELTIC each τ seconds is thus the same as the inputs over the last 8τ seconds. The DELTIC operation described above can be illustrated by the following analogy. If we record 80 seconds of conversation on tape, and then speed up the playback so that the conversation takes only 10 seconds, we have compressed the conversation just as a DELTIC would. The same

information is in the 10 second playback as was in the 80 second conversation, but it only takes 1/8th the time to hear it. We also notice that this compressed conversation has been shifted in frequency by the compression, and normal voices sound like Donald Duck. This illustrates the practical effects of the Fourier scale change theorem.

By using the DELTIC we then are able to observe the preceding 8τ seconds' worth of signal during each interval of length τ seconds. The frequency domain plots of the original signal and the output of the DELTIC show that the original spectrum has been shifted in frequency and stretched out.





The Fourier scale change theorem shows that we have multiplied the time domain characteristics of the signal by the factor 1/9, and so the frequency domain characteristics of the signal have been divided by this factor, thus effectively multiplying the frequency plot by 9.

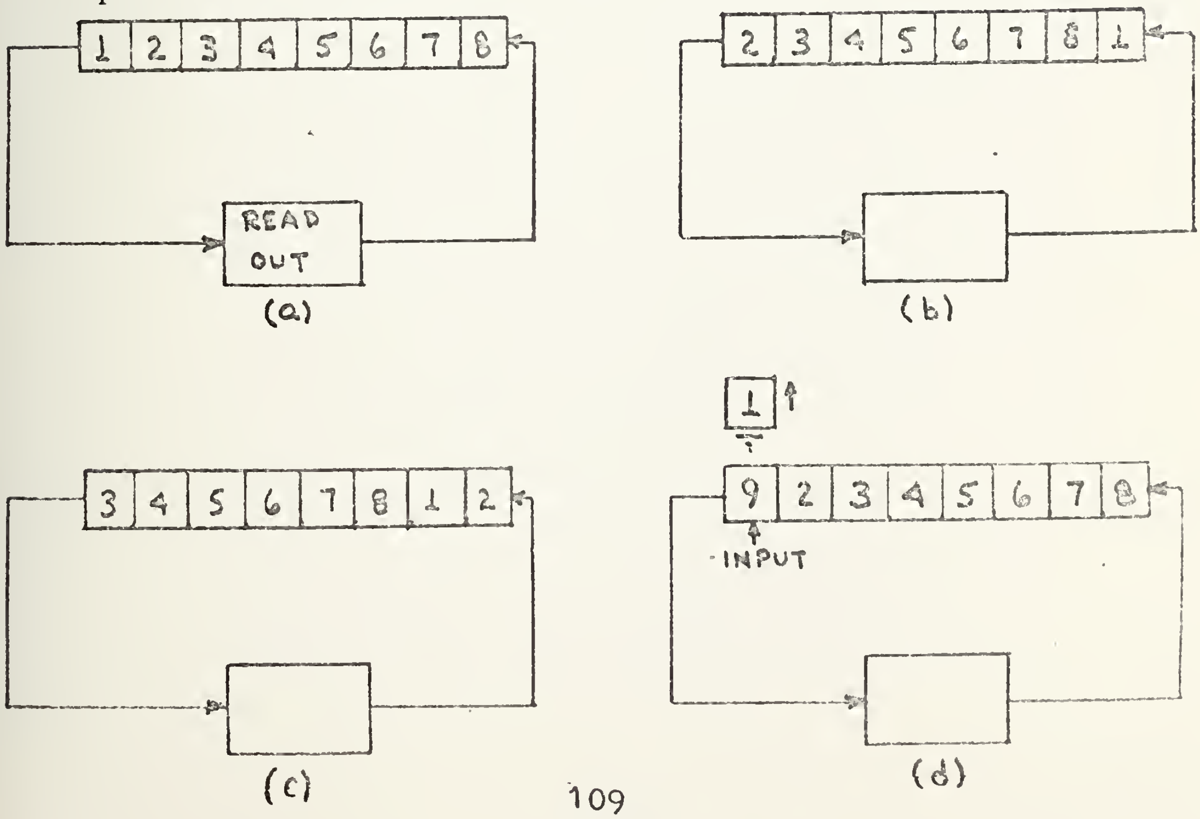
The "compression factor" is the ratio of the original sample time, T, to the compressed sample time, T'. Thus in this case it is the ratio T to T/9, or 9. Note that not only is the center frequency and bandwidth multiplied by this factor, but also the minimum resolvable frequency difference. Without compression, we would integrate for T seconds to obtain a resolvable difference of 1/T. Upon compression, this becomes 9/T. At first glance, this does not seem to help, since the resolution has not been improved. Why use the DELTIC if it doesn't help?

The benefit in using the DELTIC is that we can obtain this resolution not by integrating over T seconds, but over T/9 seconds once the DELTIC has been filled. Many spectrum analyzers "scan" the frequency band of interest by using a single band-pass filter of bandwidth equal to the minimum resolvable frequency difference of the processor, and stepping its center frequency across the band one step each integration period. Thus, if the filter bandwidth is 1/9 the bandwidth of interest, without the DELTIC it would require 9 integration periods (T) for one scan of the band. With the DELTIC, each integration

period is T/9, and thus we can scan the entire band with the same resolution in what was previously one integration time, T. This is quite an advantage for a band of several hundred hertz and a required resolution of 0.5 Hz.

The DELTIC need not be designed for insertion of a new sample value every revolution, but only a new value approximately every 10 revolutions. In this way one can compress the signal even further by having the drum revolve more than once every τ seconds.

Instead of a magnetic drum, disc, or tape memory, a digital shift register may be used. The shift register is stepped past the read-out gate in such a way as to read the entire register each τ seconds. The values are then recirculated. Each time a complete circuit is made, a new value is input to replace the oldest, and the process continues.



Some systems which use the DELTIC in processing are:

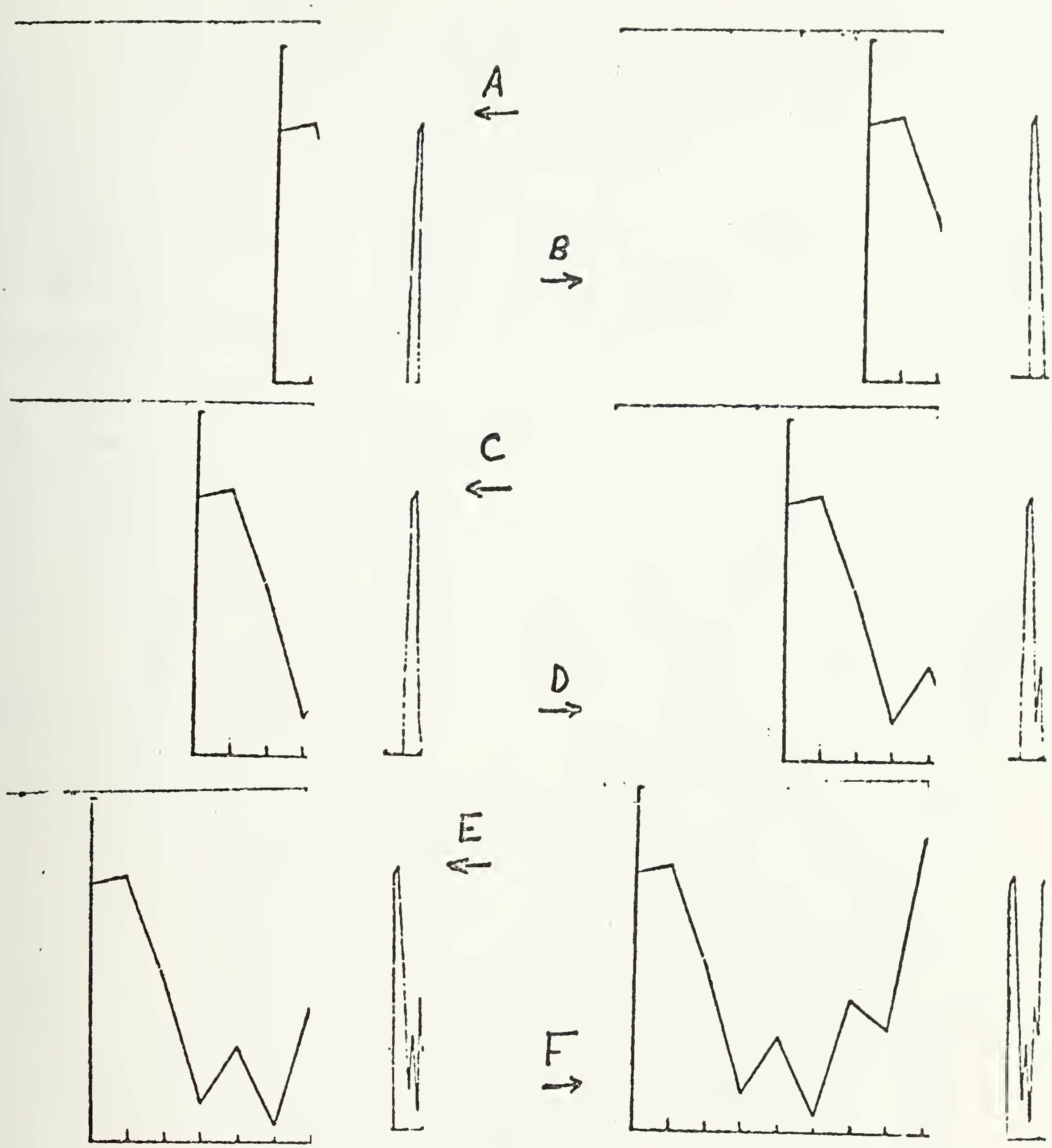
a.. AN/AQA-7

b. AN/BQR-13

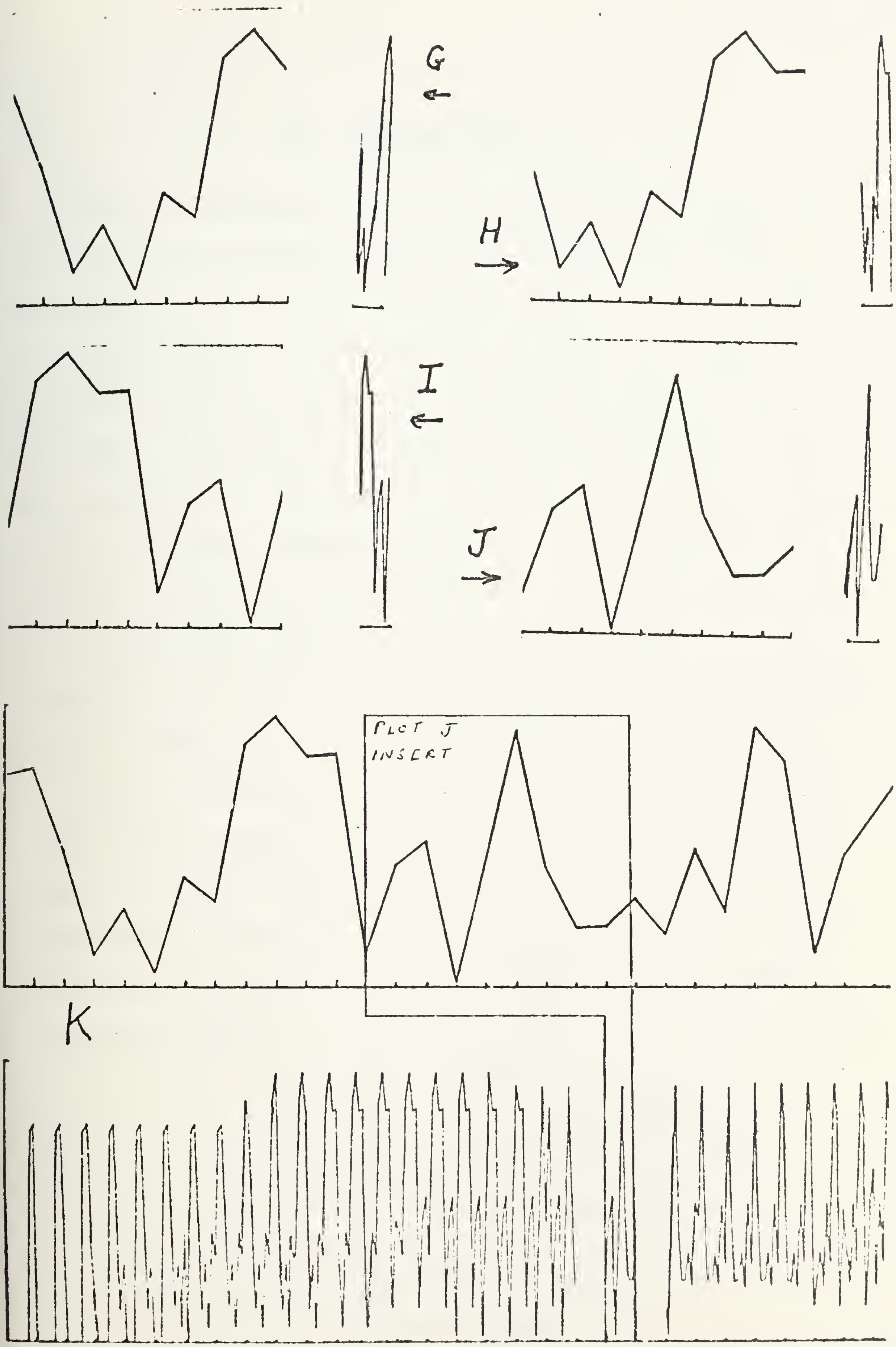
c. AN/SQS-26

d. AN/BQR-20

The illustrations on the following pages show a side by side comparison of an input signal and the resultant DELTIC output. Notice the gradual build up of the DELTIC output as time intervals of the signal are introduced. Note also that as time intervals continue to be introduced, the DELTIC drops out the oldest information and represents only the most recent time intervals.



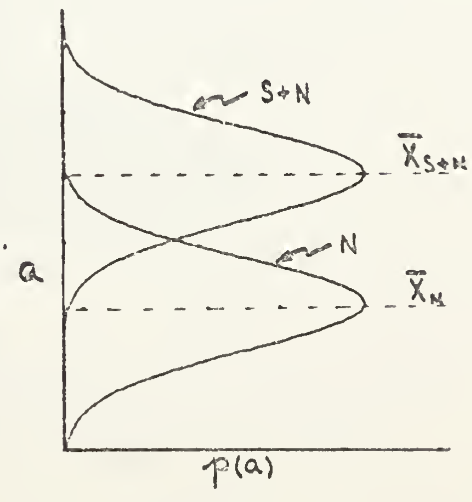
As the signal is "stepped" into the DELTIC, the output of the DELTIC represents the compressed signal. Plots A to F indicate the buildup of the DELTIC. Plots G to J show the oldest information dropping out and only the most recent information being represented. Plot K shows the complete time history of the signal and its DELTIC output.


G
H
I
J
PLOT J INSERT
K

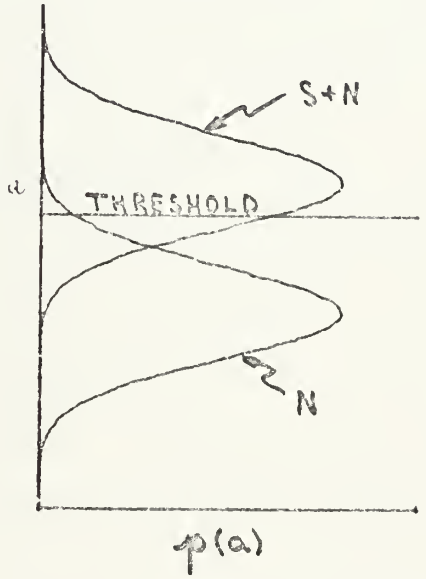
VII. ENERGY DETECTION

In the discussion of random signals it was noted that in order to apply statistical analysis to signal processing, the signals must be "ergodic". One property of ergodic signals is statistical stationarity, that is, the statistical properties of the signal are constant in time. In reality, the signals that must be processed are not strictly stationary. The sea noise varies with the time of day, changes in weather, etc., but the variation is over a long time period. This fact enables us to apply the statistical analysis approach to signal processing, as long as the processing time is short enough that the variation in signal properties is negligible.

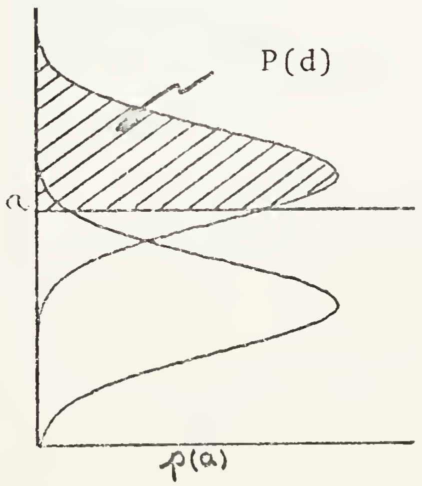
Let us consider how to apply statistical methods to detection problems. For simplicity, take the case of Gaussian noise, where the probability distributions of amplitude for noise alone and noise plus signal are Gaussian with equal variance, σ^2, but different means (average values).



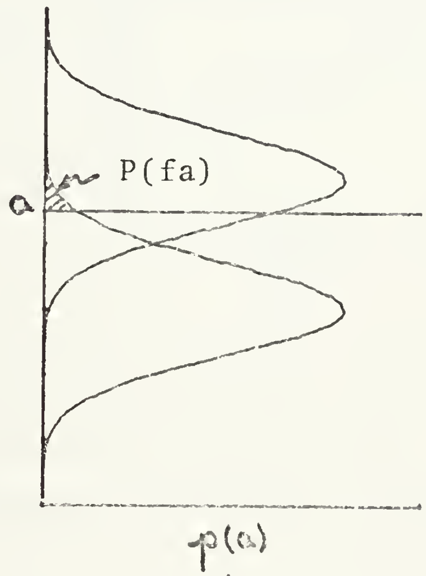
If we now have a processor which compares the amplitude of the input to a "threshold" level set into it, and indicates "signal present" if the input is above threshold, "signal absent" if the input is below threshold, then we can apply statistical theory to determine the operating characteristics of the processor.



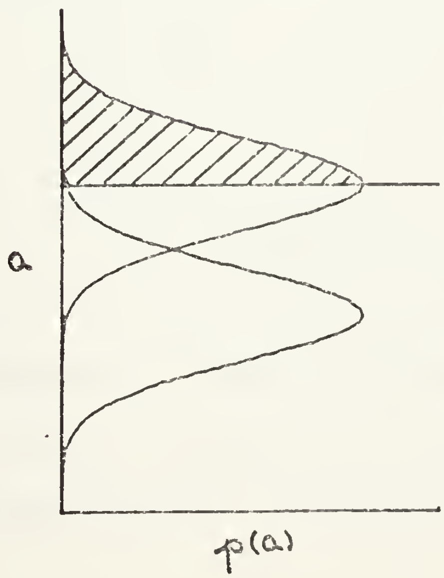
The area under the S + N curve above the threshold is proportional to the probability that the amplitude of the input will be above the threshold when the signal is present. Thus, this area is proportional to the signal detection probability, P(d).



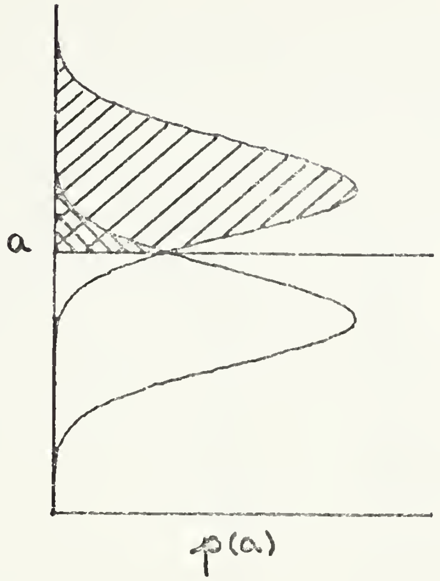
The area under the N curve and above the threshold is proportional to the probability that the amplitude of the input will be above the threshold, even when the signal is absent, because of the background noise. Thus, this area is proportional to the probability of false alarm, P(fa).



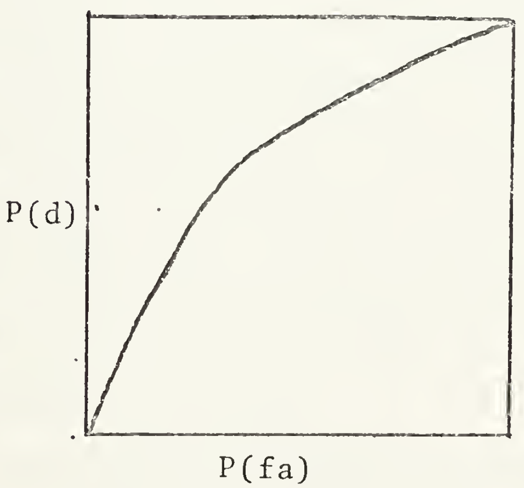
If the threshold is varied, the probabilities change. If the threshold is raised, the probability of detection decreases, but so does the probability of false alarm.



If the threshold is lowered, P(d) increases, but so does P(fa).

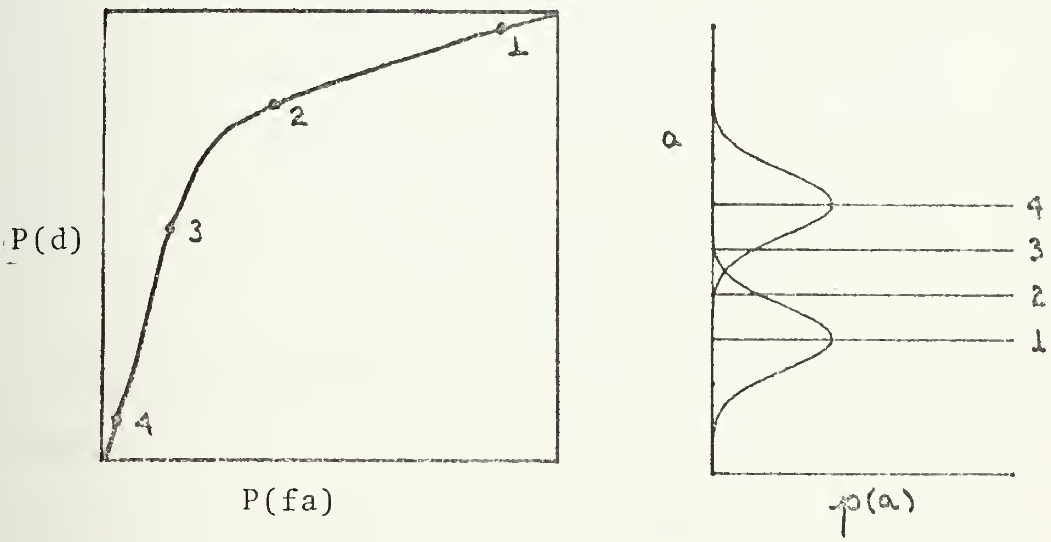


The ratio of P(d) to P(fa) does not remain constant, however. Plotting P(d) versus P(fa) as the threshold varies, we obtain the following result.



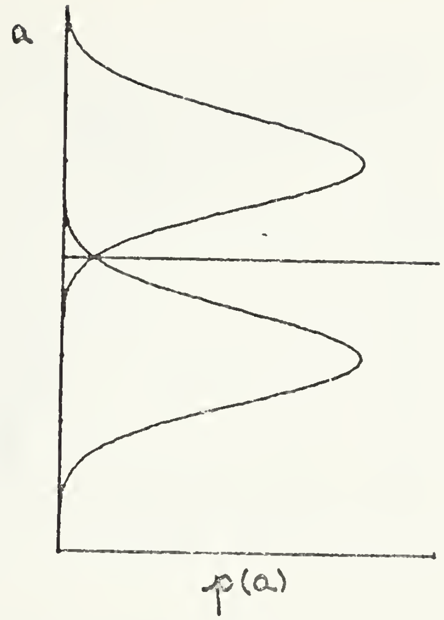
This is known as a "Receiver Operating Characteristics" curve, or ROC curve. Using this curve we can determine where to set the receiver threshold to optimize P(d) and P(fa) .

As the threshold is reised, P(d) does not change much in comparison to the rate at which P(fa) is decreasing, until the "knee" of the curve (between point 2 and point 3) is reached.

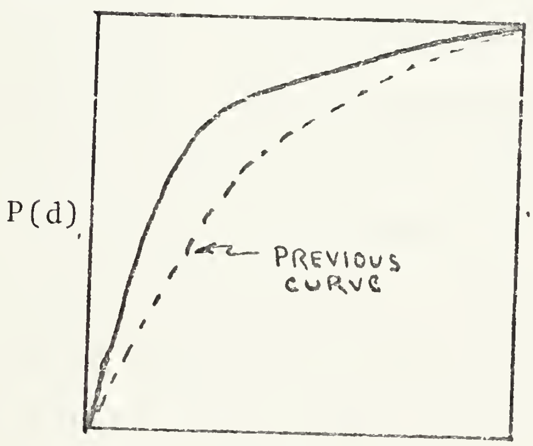


As we continue to raise the threshold above this value, P(d) starts to fall off rapidly, while P(fa) changes very little. By deciding what maximum P(fa) can be tolerated, we can set the threshold to yield the maximum P(d) possible for that particular P(fa) by referring to the ROC curve. Conversely, if a minimum P(d) is required, the ROC curve indicates the minimum P(fa) that must be tolerated.

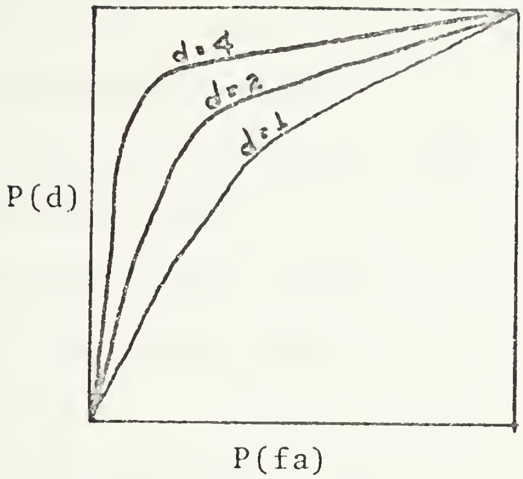
Until now the effect of changing the mean of the S + N distribution with respect to the N distribution would have on P(d) and P(fa) has not been discussed. If the mean of the S + N distribution is raised, then the ratio P(d)/P(fa) is larger for all threshold settings.



The result is shown on the ROC curve in the following diagram.



We define the "detection index" as the quantity $(\overline{X}_{s+n} - \overline{X}_n)^2/\sigma^2$, which is a measure of the difference between the distributions. A large d value indicates a large separation between the distributions, and thus a curve with more of a "knee".



This diagram shows that if the presence of a signal drastically changes the input distribution mean, then we can reduce the P(fa) significantly while still retaining a large P(d). If the presence of a signal does not drastically change the mean, large values for P(d) are possible only at the expense of a large P(fa).

We have discussed thresholds, and the way the selection of threshold values affects the probability of detection and the probability of false alarm, but have not discussed how these thresholds are set. One point that may not be readily apparent is that in detection systems in which the human operator is involved in the decision as to presence or absence of a signal, his perception of the display must be included in computation of the threshold value. Most systems in use operationally involve an operator in the decision process. The SQS-23 sonar is an example. The operator can vary the threshold to an

extent by adjusting the brightness and intensity of the display and the gain control, but a large consideration in determining whether or not a target is present is his ability to distinguish between target and noise as presented on the PPI scope.

The operator reading grams on an AQA-7 is another example. He can control the gain, or the grey value of the background on the paper, but his perception of the distinction between target and noise determines the threshold. For this reason it is difficult to determine the ROC curves for systems with human operators, because it is difficult to determine the actual threshold setting. The threshold will vary from day to day, or even hour to hour, with the same operator and the same hardware settings because of the operator's state of mind, physical condition, fatigue, etc. This is what makes quantification of the detection characteristics for a system difficult. Completely automatic systems are much easier to evaluate, as one can determine exactly what the settings are.

In the preceding discussion of energy detection and thresholding we have seen how a system's detection capability can be determined from knowledge of the characteristic distributions of noise and signal plus noise. Implicit in this treatment is the assumption that the distributions of noise and signal plus noise are exactly known. If these distributions are exactly known we can specify the performance of the system for any given threshold setting

in terms of P(d) and P(fa) . If we do not know these distributions exactly, our analysis will not exactly describe the system performance. Thus, the accuracy of our prediction of system performance depends on the accuracy with which we can describe these distributions. In designing real world systems this is one of the problems facing the designer,since the distributions of noise and signal plus noise are not constant in time (over any lengthy period) or with geographical position. For this reason many systems do not always perform as well as the specifications indicate. Only if the ambient conditions are the same as those for which the system was designed will it perform as predicted.

VIII. CORRELATION DETECTION

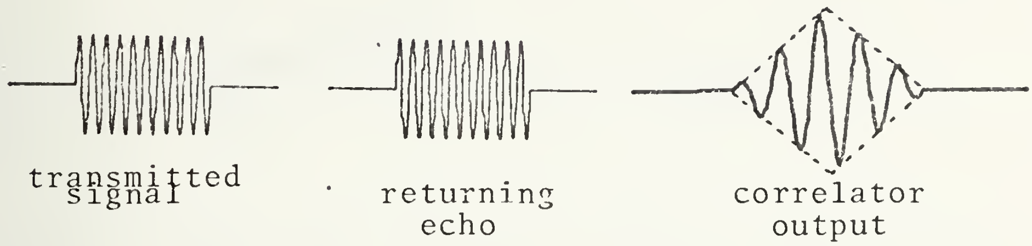
In the section on random signals, power spectral density, and noise, it was shown that correlation can effectively improve the signal-to-noise ratio for correlated signals due to the cross product terms in the calculation of the correlation function. How can this information be used to detect signals in noise? Let us study an active echo ranging system as an example.

In this type of system a signal is transmitted, and then the returning signals are processed. The range of the reflecting object can be determined from the echo return time. In a noise-free environment, there are few problems in this system, but in most real situations the ambient noise may "bury" some of the returns. By using correlation, we can improve the signal-to-noise ratio and recover these echoes.

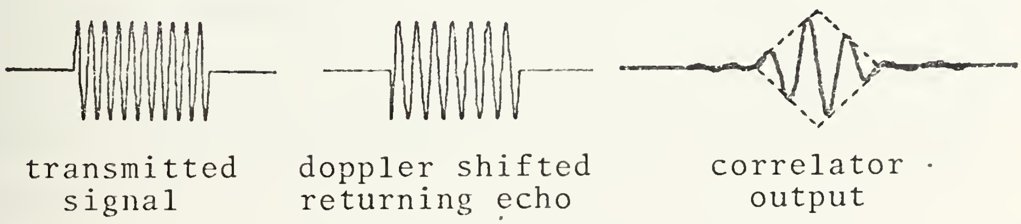
This system retains a replica of the transmitted signal and then inputs the received signal into the correlator. The output of the correlator is large when an echo is received, and small when only noise is present.

What does doppler do to the correlation of returning echoes? Let us look at a burst of CW signal transmitted, and the effect of doppler shifting on the correlation function. If the echo is not doppler shifted, the output of the correlator will be the autocorrelation of the

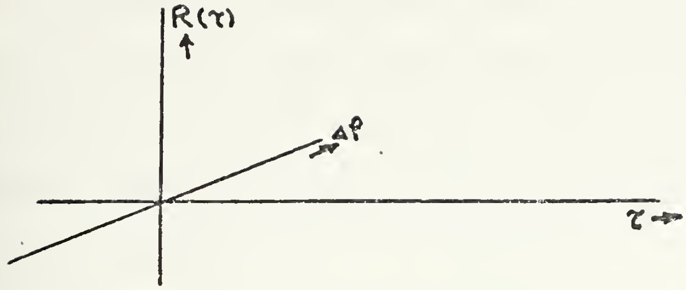
transmitted signal at a time , which is related to the range of the reflected object.



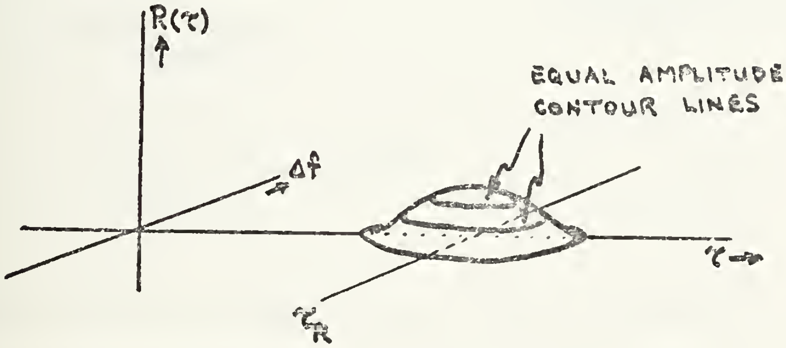
As the returning echo is shifted in frequency by doppler, the output of the correlator becomes the cross-correlation of two closely related signals, but its value is never as high as the autocorrelation. The cross-correlation amplitude and the amount of delay over which it is correlated decrease. (In fact, the reduction results from the decrease in the delay over which the signal is correlated with its doppler shifted echo.)



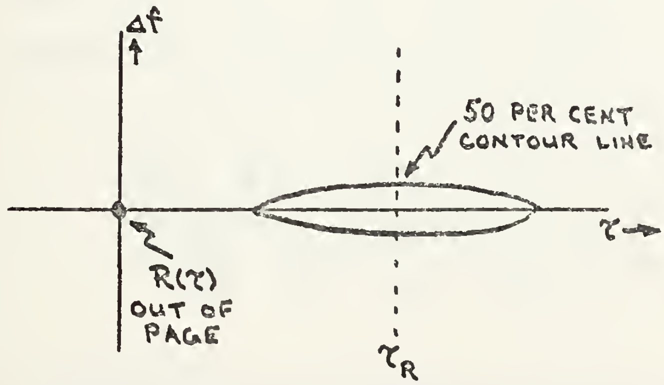
A plot of the correlator output for a number of different doppler shifts on the same diagram, that is, a τ, Δf, correlator amplitude plot, shows the effects of doppler.



For a given echo, this plot gives a solid surface relating Δf, τ, and amplitude.

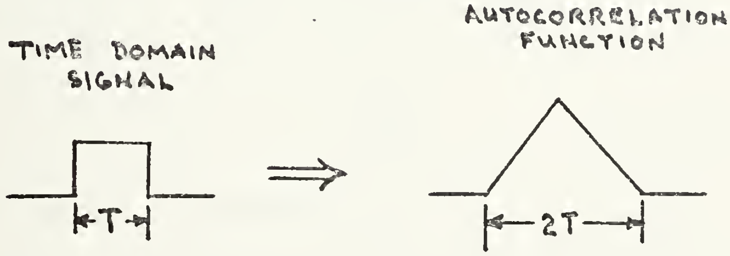


If we look down at the τ, Δf-plane, and plot the contour line where the amplitude of the function is 50 per cent of the maximum, we obtain the following result.

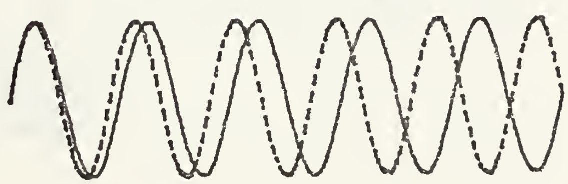


This diagram is commonly known as an "ambiguity diagram", and the reason for this will become apparent from examination of FM pulses. First, however, let us consider a long CW pulse to see how doppler affects the correlator output.

The section on correlation and convolution shows that the width of the correlation function is related to the duration of the time-domain signal.

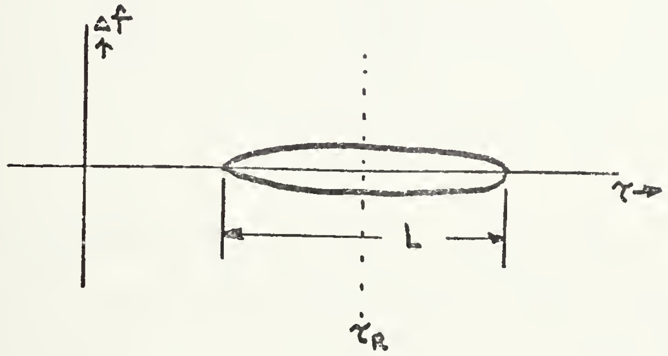


Thus, for a long CW pulse the correlator can be expected to respond over a long range of values of .
What about doppler and its effect on a long CW pulse being correlated? Observation of two signals at slightly different frequencies shows that the longer the pulses to be correlated, the less the correlation, since the signals will be almost correlated over only a few cycles, but get further and further out of phase as time progresses.



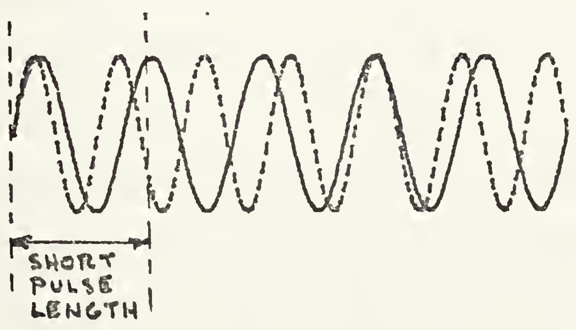
As the difference in frequency between the signals increases, the number of cycles over which the two are

almost correlated decreases rapidly. Thus for long CW pulses, the correlator response decreases rapidly with doppler shifts. The ambiguity diagram for a long CW pulse is shown in the following figure.

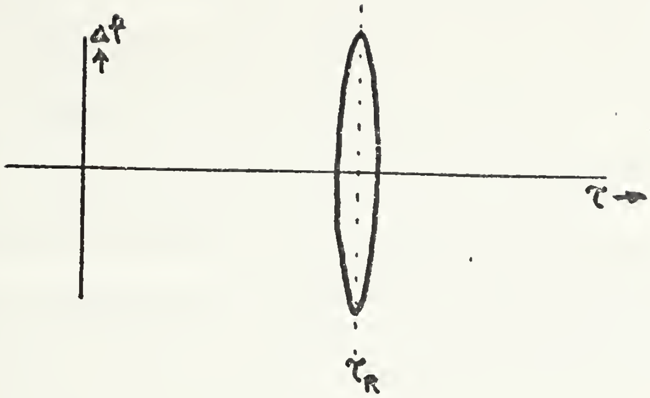


The length L is proportional to the length of the CW pulse. τ_R is the delay corresponding to the actual range of the target.

For a short CW pulse, the length of the correlation peak 50 per cent contour will be shorter, but the range of doppler shifts over which the signals remain essentially correlated is greater.



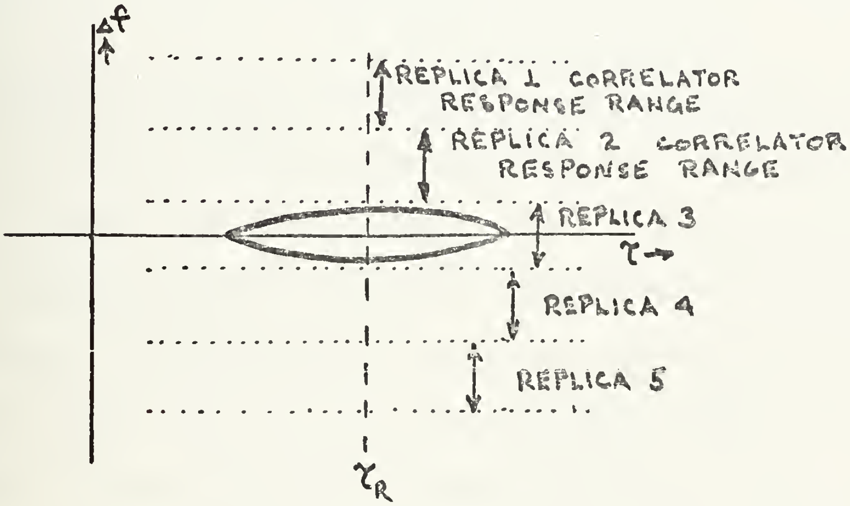
Thus the ambiguity diagram for the short CW pulse has the following appearance.



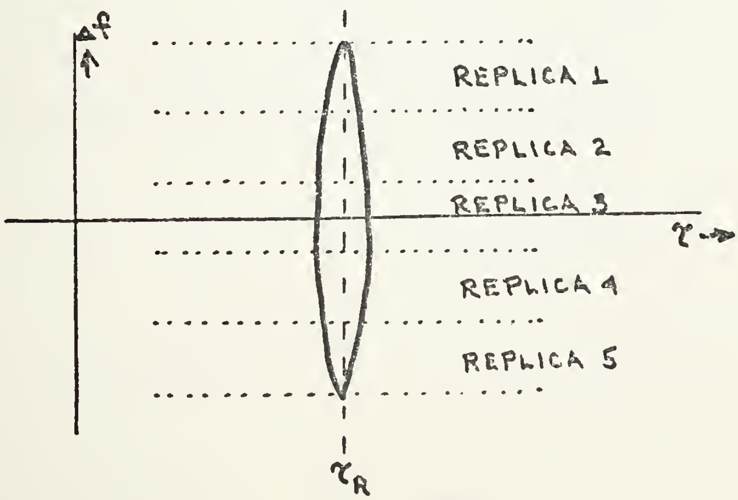
These two diagrams show why long pulse CW is good for searching and doppler determination, and short pulse CW is good for range resolution. A sonar pulse of 120 milliseconds will ensonify a region of water 180 meters long. In other words, the spatial length of the pulse as it travels through the water is 180 meters (1500 meters/second velocity x 0.120 seconds = 180 meters, the distance the forward edge of the pulse has traveled during the time the rest of the pulse was being generated). The correlation peak 50 per cent contour, as shown on the ambiguity diagram, will be proportional to the pulse length T, and for CW this turns out to be a one-to-one ratio. Thus, the correlation will be over a period of 120 milliseconds, which means that the target range can be determined only to the nearest 180 meters.

In the case of the short pulse of, say 5 milliseconds, the pulse spatial length is 7.5 meters, the correlation peaks over 5 milliseconds, and the range resolution is 7.5 meters.

The effect of doppler on the ambiguity diagram shows that the same type of development can be made to relate frequency resolution of the two different pulse lengths. Because the long pulse remains correlated over very limited doppler shifts, the frequency of the returning echo can be determined rather accurately by using a bank of correlators comparing the echo to replicas with varying amounts of doppler shifts.



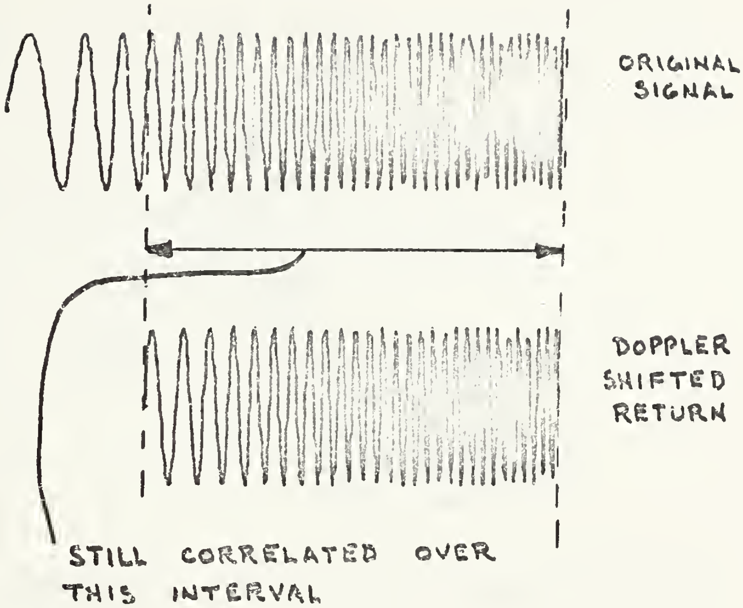
On the other hand, use of the same bank of correlators with a short pulse produces a response from each of these correlators, because the short pulse remains correlated over a large range of doppler shifts.

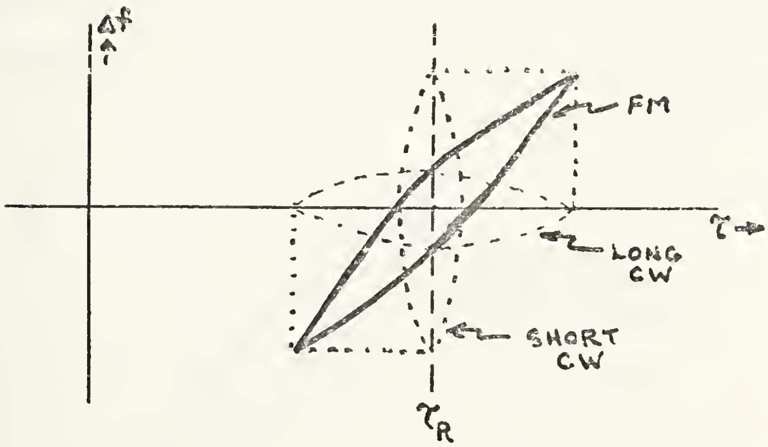


If we use a pulse in which the frequency is changed during transmission, which is commonly referred to as FM slide, several differences appear.

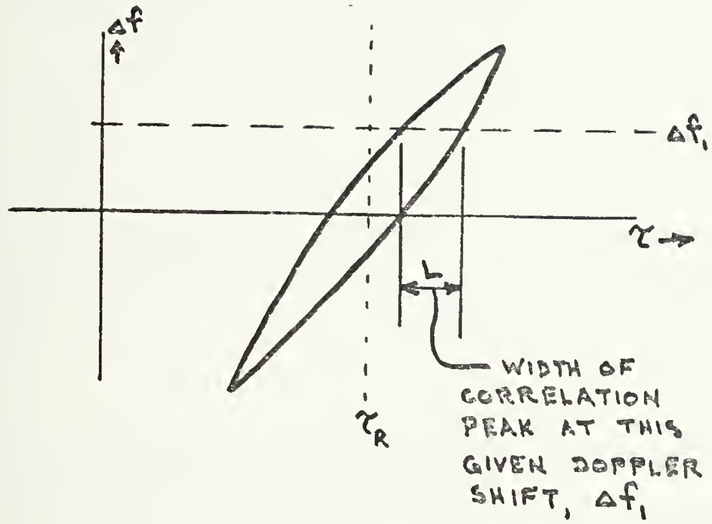
FM "Up-Slide"

Because of the frequency change during the pulse, the returning echo correlates with the transmitted replica over a short range of time, τ, even for long pulses. Thus, the ambiguity diagram is very narrow in the τ direction. On the other hand, if a long pulse is used, the Δf range of correlation is narrow. One difference between FM and CW pulses becomes apparent here, however. In the case of FM pulses, a doppler shift will tend to shift the entire pulse in frequency, but the slide will remain linear (provided it was originally linear), and thus it will still tend to correlate but at a different value of τ. This behavior and its effect on the ambiguity diagram are shown in the next drawing.

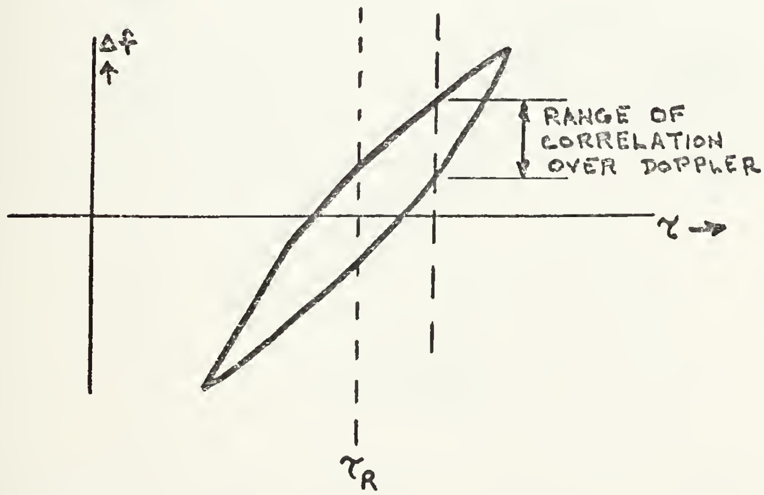




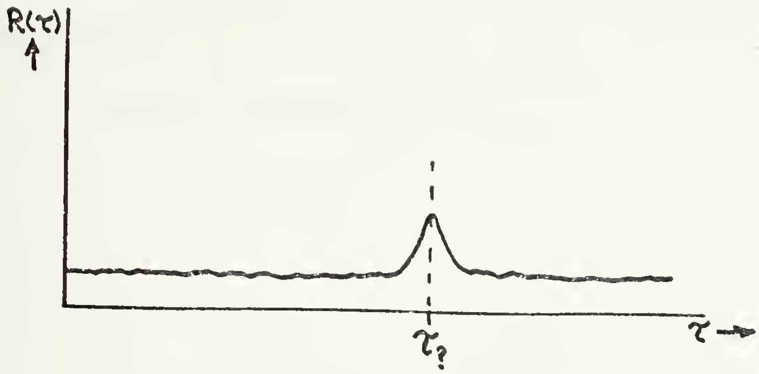
Note that the FM slide pulse has some of the characteristics of both long and short pulse CW. At any given doppler shift, the correlation peak itself is narrow, as is the peak for short CW pulses. Thus, range resolution is good.



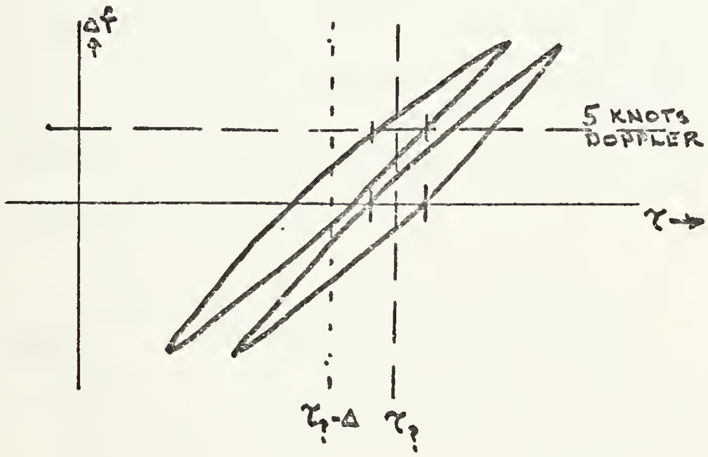
Note also that at a given τ the range of doppler shift over which the signal is correlated is narrow, on the order of the range over which the long CW pulse is correlated.



The reason this diagram is called an "ambiguity diagram" can now be seen. The correlator output, that is the τ-amplitude plot, shows the following.

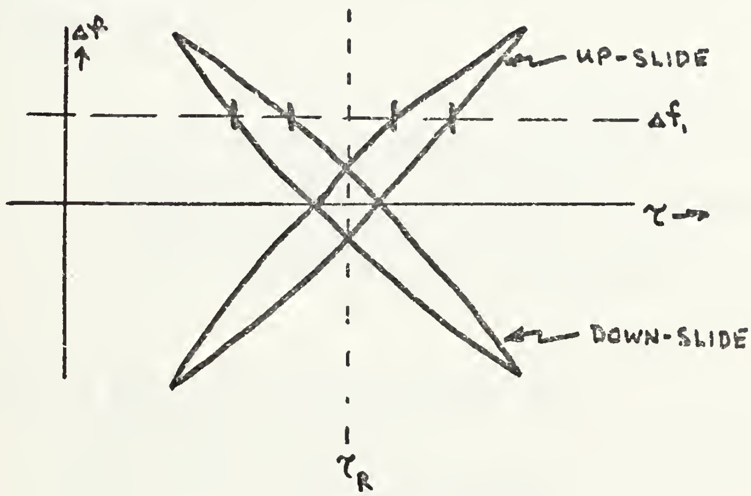


Was the peak at $\tau_?$ caused by a target exhibiting no doppler at range equivalent to $\tau_?$, or was it caused by a target with 5 knots of doppler at range $\tau_? - \Delta$? The output looks the same in either case. There is a "range ambiguity" present.

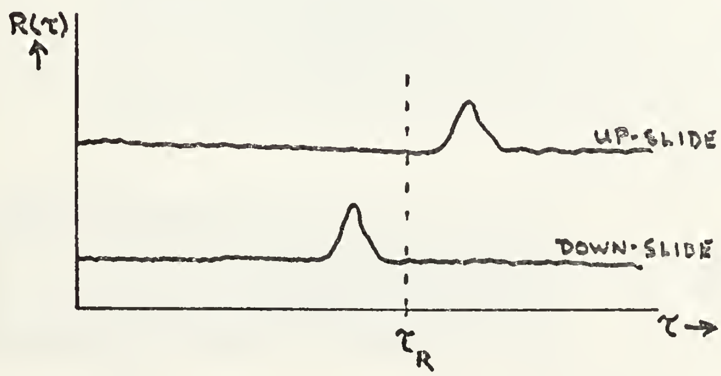


One way of resolving this is to transmit two FM slide pulses, one with an "up-slide", (increasing frequency with time), and one with a "down-slide"(decreasing frequency with time).By then using two correlators, one with an

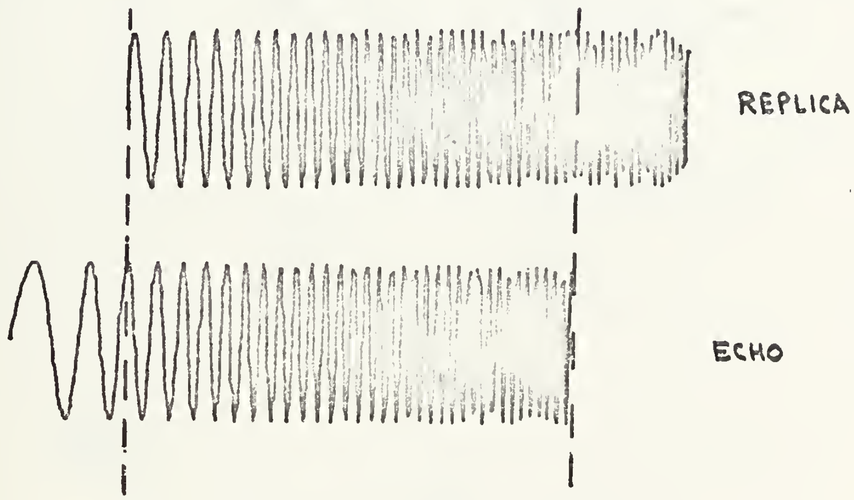
"up-slide" replica and one with a "down-slide" replica, the ambiguity can be resolved. The next ambiguity diagram shows the output from both correlators for a given range target.



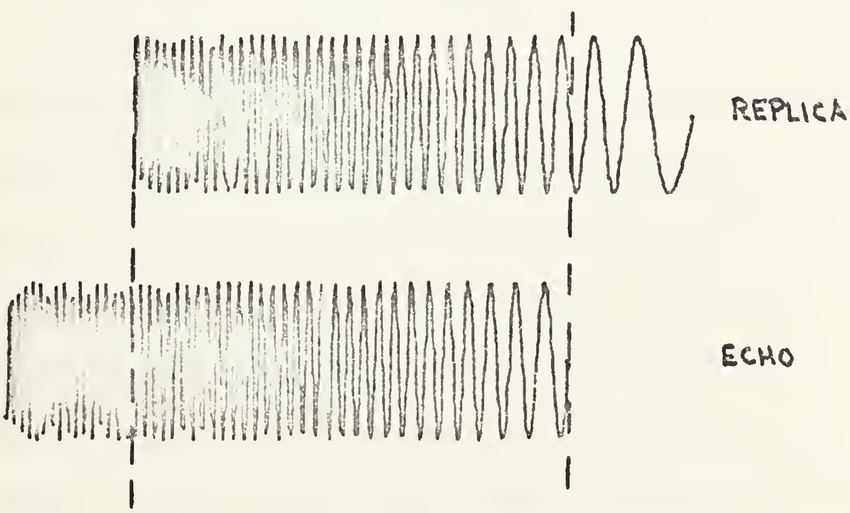
When compared on a τ-amplitude plot, the correlator outputs for a target exhibiting the doppler shift shown on the ambiguity diagram by the dotted line appear as in the following diagram.



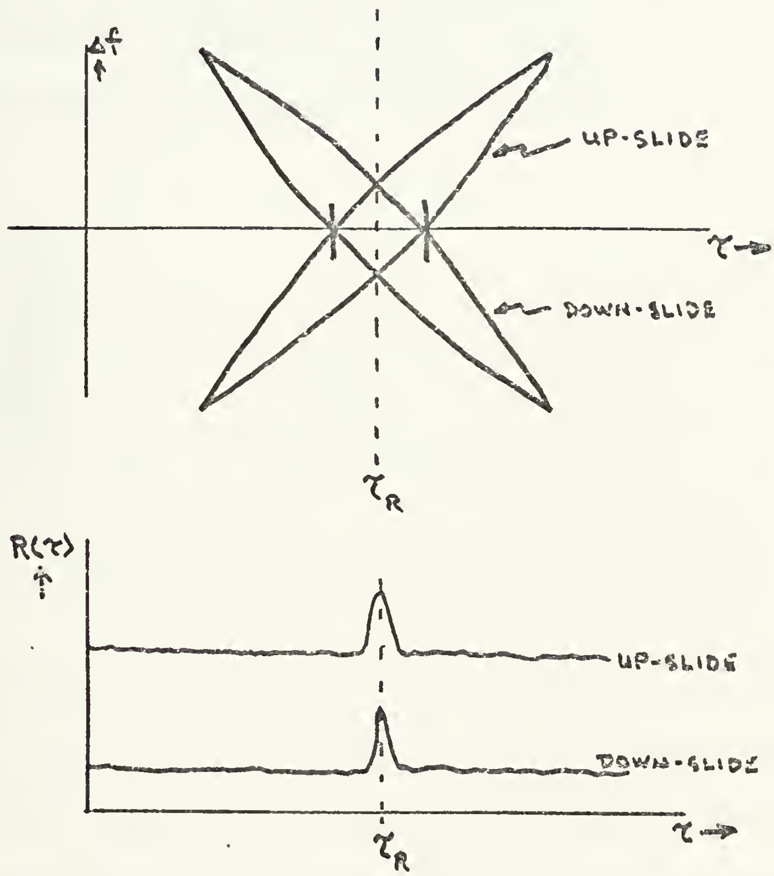
By noting which peak appears first, we can determine if the doppler is up or down. If the "up-slide" peak appears first, then the returning echo has been shifted down in frequency, and thus correlates at a delay, , earlier than it would if not doppler shifted. This indicates <u>down</u> doppler.



If the "down-slide" peak appears first, the opposite is true, which indicates <u>up</u> doppler.



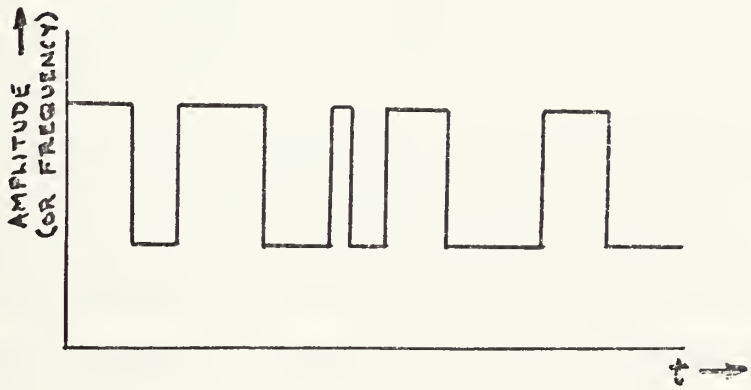
The true range of the target is midway between the two correlation peaks, regardless of the amount of doppler shift present. If the target has no doppler, the two peaks coincide.



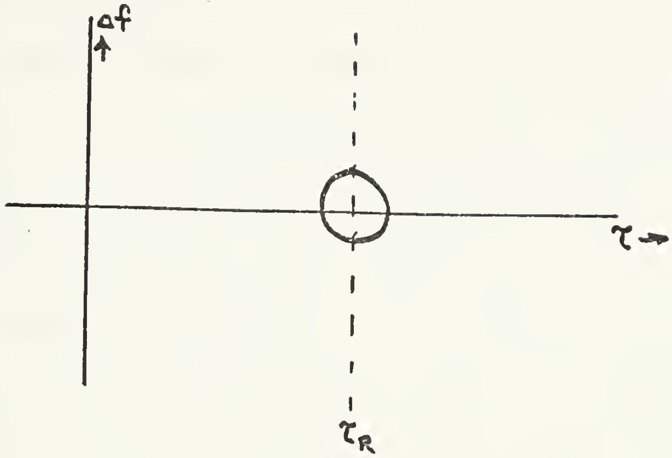
The separation between the up-slide and down-slide correlation peaks is proportional to the amount of doppler shift in the returning echo. If a suitably calibrated display of the two outputs is available, the amount of doppler may be read directly from the presentation.

Thus, by employing FM slide pulses and suitable processing, one can obtain the range resolution of short CW pulses and the doppler discrimination of long CW pulses from a pair of long FM pulses.

Another type of pulse which provides the "best of both worlds" with respect to doppler and range is known as the "pseudo-random noise pulse", which consists of a "coded" pulse with amplitude or frequency varied in a pseudo-random fashion. Take, for example, the case where the amplitude (or frequency) of the signal is not a simple sinusoid, but a binary coded function.



This pulse is correlated over a very narrow range of τ, giving good range resolution, and it is also correlated over a very narrow range of doppler shift because of its pseudo-random character (Truly random signals correlate only at a single value of τ.) The ambiguity diagram for a pseudo-random pulse is shown in the next figure.



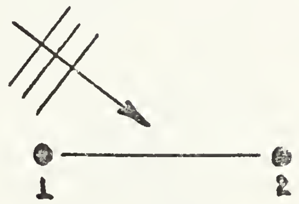
Note that because of the correlation over narrow ranges of Δf and τ, no range ambiguity is present. The problem associated with this type of pulse, however, is that the presence of noise in the returning echo causes marked degradation in the correlation of the return, and thus degrades the performance of real systems. For this reason the pseudo-random noise pulse is useful in applications where the signal-to-noise ratio is high, such as close range tracking, bathymetry, etc., but its usefulness is limited in long range search and tracking, where the signal-to-noise ratio is much lower.

Up to this point we have covered the use of correlation in active systems, and how the use of FM slide pulses combined with the proper correlation processor enables us to determine the amount of doppler in an echo directly. Let us now examine some applications of correlation in the passive detection field.

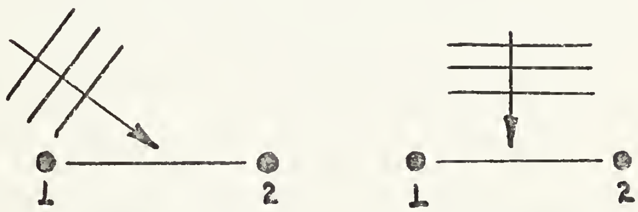
One difference between active and passive correlation detection systems becomes apparent very quickly. That is, in the active system we compare the returning echo with a replica of the transmitted pulse. In other words, the signal we are seeking is a known quantity (a replica of the transmitted pulse). In a passive system, however, no pulse is transmitted, therefore we have no replica for comparison with the incoming signal. Therefore we cannot utilize correlation in the same manner; however it is still of use, particularly in passive direction-finding systems. The utility of passive direction-finding systems can be seen by examination of a typical system and its operation. Consider a system composed of two hydrophones, separated in space by some distance, d.

If the output of hydrophone 1 is used as a replica with which to compare the output of hydrophone 2, the cross-correlation of the two outputs can be obtained. What does this do for us? The important property of sea noise for this system is that it tends to be isotropic, that is to be the same regardless of the direction from which it impinges on the hydrophone. In addition, sea noise tends to be uncorrelated over distance. On the other hand, most signals of interest to us are from a single source, usually a submarine, and thus are coherent to an extent, and directional in nature.

Since sea noise is isotropic, the correlator output will be small and relatively constant when no signal is present. Now consider what happens in the presence of a signal of interest. The acoustic energy traveling outward from the source will impinge on the hydrophones from some given direction.



Following a given wavefront as it travels across the hydrophones, we see that it will encounter one of the hydrophones before the other unless the direction of impingement is perpendicular to the line joining the two hydrophones, or the baseline.

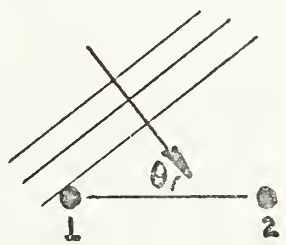


The difference between the times that the wavefront strikes the two hydrophones depends on the direction from which it arrives with respect to the baseline. If the direction of arrival is perpendicular to this baseline, the wavefront strikes both hydrophones at the same time, and thus there is no time difference. If the direction of arrival is parallel to the baseline, the

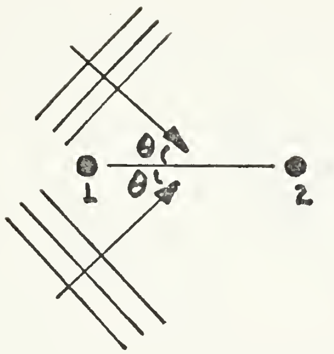
time difference is a maximum. In fact, the time difference is related to the arrival direction by the following equation:

$$\Delta t = \frac{d}{v \cos \theta},$$

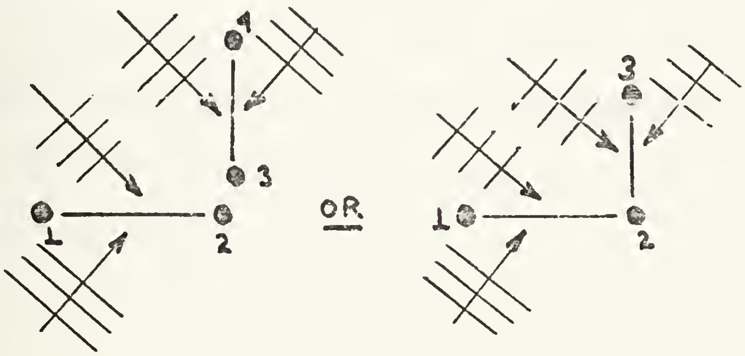
where v is the speed of sound in the medium, and θ the wavefront arrival angle.



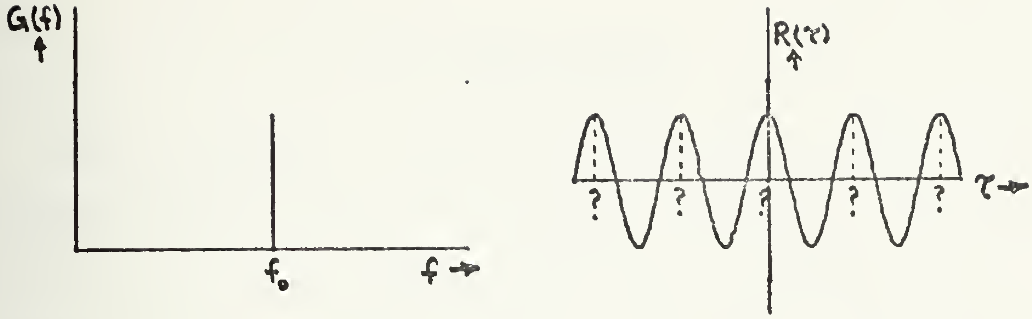
The cross-correlation of the two hydrophone outputs for a single wave passing the array, shows that the delay time τ at which a correlation peak appears is a function of the wavefront arrival time difference at the hydrophones. If the wavefront arrival angle is 90°, then the arrival time difference is zero, and thus the correlation peak will appear at $\tau = 0$. For any other arrival angle, there is a positive arrival time difference, Δt, and the correlation peak appears at $\tau = \Delta t$. This allows us to relate the correlation delay time, τ, to the wave arrival direction with respect to the array baseline. There is a problem of directional ambiguity, as arrivals from opposite sides of the array at equal angles with respect to the baseline produce the same delay.



This ambiguity can be resolved by adding another array to cross-fix, or by adding a third hydrophone not in line with the first two, to resolve the ambiguity.



It should be noted that although this method works well with broadband radiated signals, it has some drawbacks when applied to signals of a single frequency line or those with a very broad autocorrelation function. The problem is as follows. If a signal consists of a single frequency line, its autocorrelation function is a sinusoidal function. The question then arises as to which peak corresponds to the signal arrival direction.



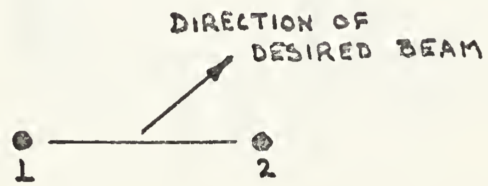
For this reason, a correlation detector is not well suited for passive detection of narrow-band signals, but works well for broadband signal detection.

We have now seen how correlation can be of help in active systems and in passive broadband signal detection systems. What can be done about single-frequency line detection? In order to answer this question, we examine a system which is similar to correlation in many aspects, but is not actually correlation.

IX. BEAM FORMING

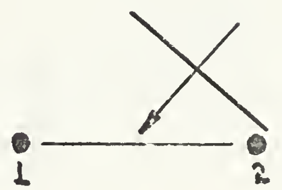
Beam forming, or electrical steering of arrays, is accomplished by a process which is similar to correlation, but with one major difference--it uses summation, vice multiplication, of the outputs. We examine the process to see how it operates.

Let us start with a two-hydrophone array. As noted in the preceding section, the acoustic wave arrival direction determines the wavefront arrival time difference at the hydrophones. With this in mind, suppose that it is desired to "steer" the array to receive signals from a given direction relative to the array baseline selectively, that is, to form a "receiving beam" at a given orientation.

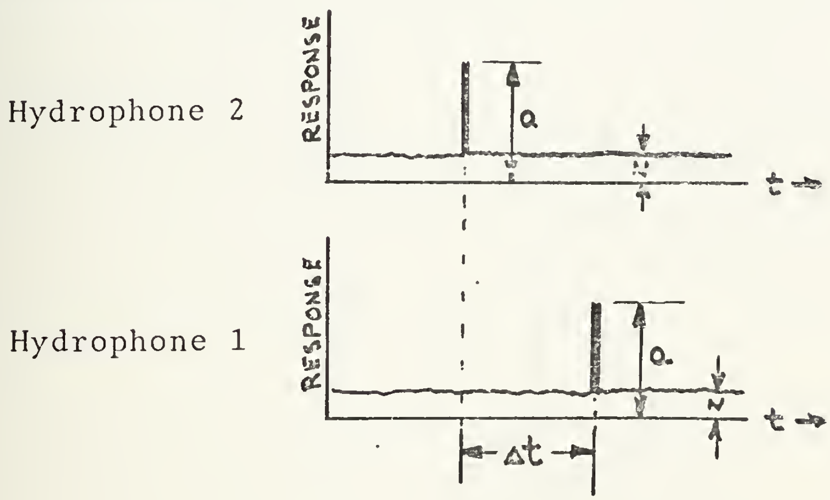


If we delay the output from hydrophone 2 for a time $t_d = \Delta t$, and add t_d to the time necessary for the acoustic wave from the desired direction to travel from hydrophone 2 to hydrophone 1, we can selectively bias the array to receive signals from that direction.

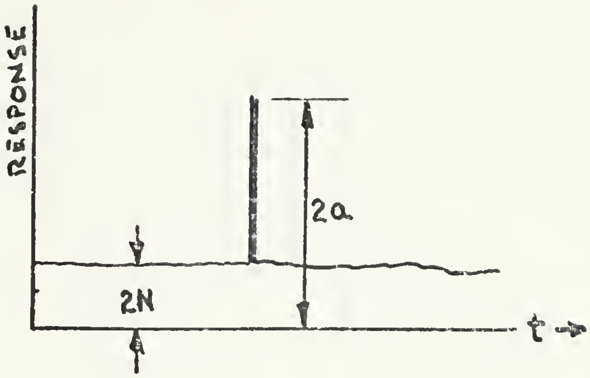
Why does this selectively bias the response of the system to signals from that direction? Consider a single impulse arriving from the direction of our selected beam.



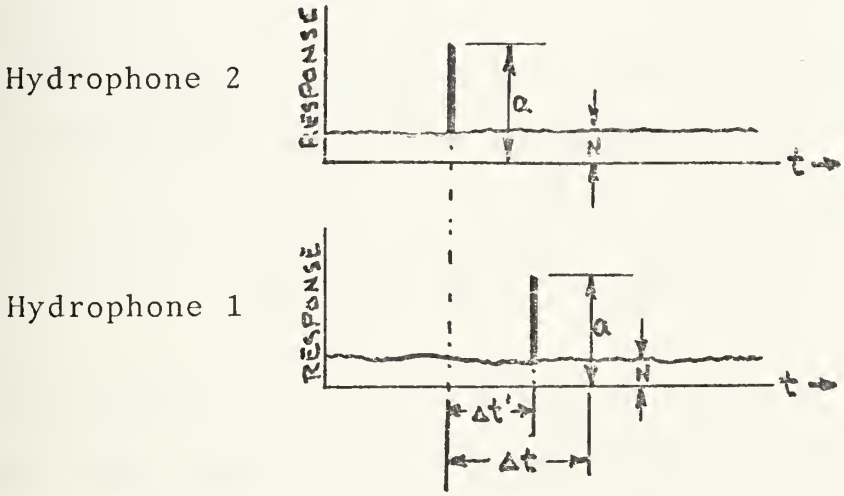
Plotting the output of each hydrophone as a function of time produces the following result.



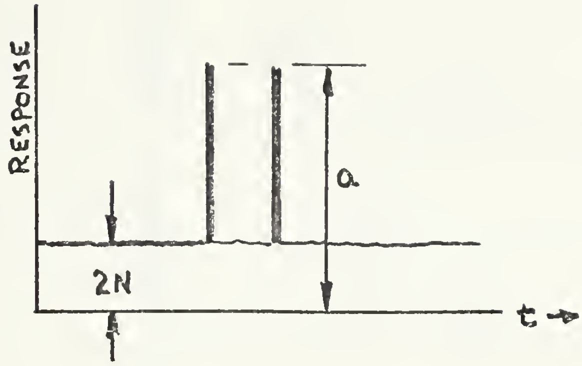
By delaying the output of hydrophone 2 for a time $t_d = \Delta t$ and adding the output of the two hydrophones, the two impulses reinforce each other and the output will appear as in the next drawing.



If, however, the impulse arrived from a different direction, the time difference between arrival at the two hydrophones will be different.

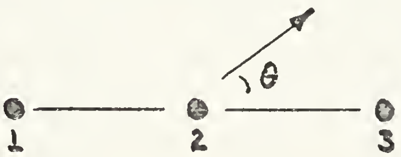


If we input the output of hydrophone 2 through the same delay, Δt, the plot of the summation of the two signals is as shown in the next illustration.



Thus we see that signals arriving from the selected direction (time delay Δt) will selectively reinforce one another in the processor while those from other directions will not.

If we examine a 3-hydrophone linear array we observe that the effect is even more pronounced.

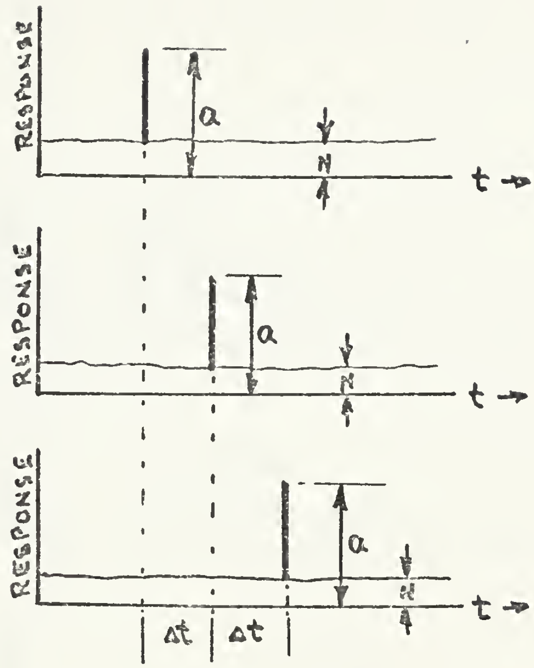


By delaying the output of hydrophone 3 for a length of time $2\Delta t$, and the output of hydrophone 2 for a length of time Δt, a wave from the desired direction will produce the effect shown in the next figure.

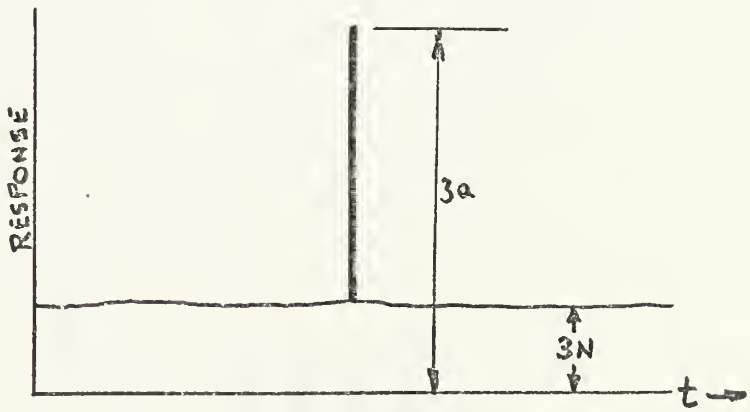
Hydrophone 3

Hydrophone 2

Hydrophone 1



Output after
summation

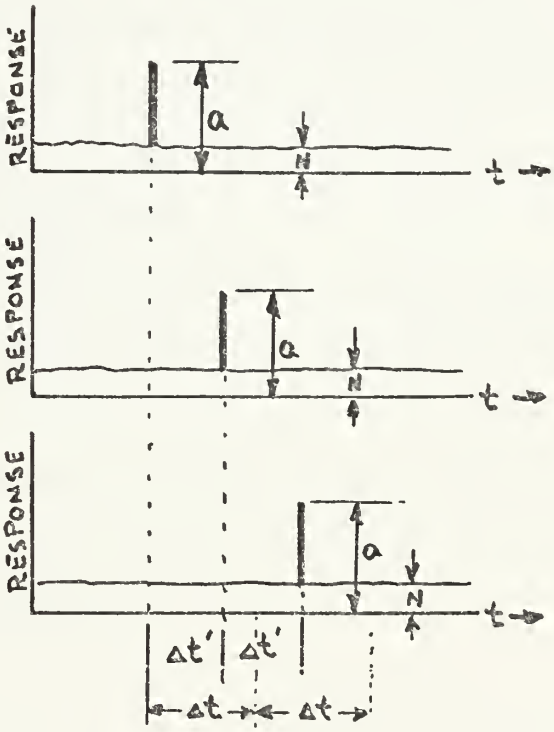


A wave from a different direction will give the following output.

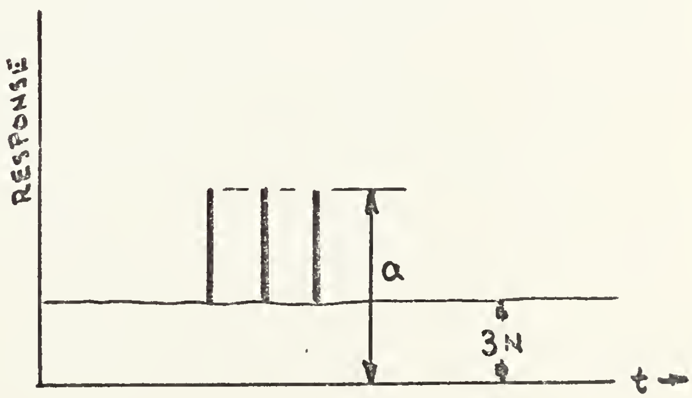
Hydrophone 3

Hydrophone 2

Hydrophone 1



Output after summation



It is apparent that adding hydrophones to the array makes it more responsive to signals from the chosen direction. Also, since the output of the system is not a correlation of the individual hydrophone outputs, but a simple summation of their energies, the system will work for narrow-band or single-frequency line signals as well as for broadband signals.

By using a number of hydrophones in this manner in conjunction with an energy detector, and by setting the threshold high enough to preclude response to the unreinforced signals from other directions, one has a very directional detection capability from a series of omnidirectional hydrophones.

BIBLIOGRAPHY

Bracewell, R., The Fourier Transform and Its Applications, McGraw-Hill, 1965.

Carlson, A. B., Communication Systems, McGraw-Hill, 1968.

Decker, T. R. and Sollenberger, R. T., The Environment and Its Effect on Antisubmarine Warfare, Masters Thesis, Naval Postgraduate School, Monterey, 1975.

King, L. F. Jr. and Swift, D. A., Acoustic Primer for Underwater Sound Applications to Antisubmarine Warfare, Masters Thesis, Naval Postgraduate School, Monterey, 1975.

Sakrison, D. J., Communication Theory: Transmission of Waveforms and Digital Information, Wiley, 1968.

Smith, B. J. and Vice, L. W., Probability and Its Applications to Antisubmarine Warfare, Masters Thesis, Naval Postgraduate School, Monterey, 1975.

Taub, H. and Schilling, D. L., Principles of Communication Systems, McGraw-Hill, 1971.

Urick, R. J., Principles of Underwater Sound for Engineers, McGraw-Hill, 1967.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Chairman, Code 71 Antisubmarine Warfare Group Naval Postgraduate School Monterey, California 93940	2
4.	Assoc. Professor G. L. Sackman, Code 52 Sa (Thesis Advisor) Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	1
5.	LT William Oris Davis, USN (Student) Class 49 Naval Destroyer School Newport, Rhode Island 02840	1
6.	LT Paul Howard Donaldson, USN (Student) c/o Lt. Col. Floyd Herbert, USAF (Ret.) P. O. Box 523 Athens, Alabama 35611	1